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TEST BOOKLET

Sl. No: 0461

Subject Code: 15 | **Subject: MATHEMATICS**

WRITTEN TEST FOR RECRUITMENT OF POST GRADUATE TEACHERS FOR NON-GOVT. AIDED HIGHER SECONDARY SCHOOLS OF ODISHA

Time Allowed : 2 Hours | *Maximum Marks: 150*

: INSTRUCTIONS TO CANDIDATES:

1. **IMMEDIATELY AFTER THE COMMENCEMENT OF THE EXAMINATION, YOU SHOULD CHECK THAT THIS TEST BOOKLET CONTAINS 20 PAGES AND DOES NOT HAVE ANY UNPRINTED OR TORN OR MISSING PAGES OR ITEMS ETC. IF SO, GET IT REPLACED BY A COMPLETE TEST BOOKLET.**

2. You have to enter your **Roll No.** on the Test Booklet in the Box provided alongside. **DO NOT** write anything else on the Test Booklet.

3. The Test Booklet contains **100** questions. Each question comprises four options. You have to select the correct answer which you want to mark (darken) on the OMR Answer Sheet. In any case, choose ONLY ONE answer for each question. If more than one answer is darkened, it will be considered wrong.

4. You have to mark (darken) all your answers only on the **OMR Answer Sheet using BLACK BALL POINT PEN** provided by the State Selection Board. You have to do rough work only in the space provided at the end of the Test Booklet. See instructions in the OMR Answer Sheet.

5. All questions carry equal marks. While **1.5** mark will be awarded for each correct answer, each wrong answer will result in negative marking of **0.50** mark.

6. Before you proceed to mark (darken) the answers in the OMR Answer Sheet to the questions in the Test Booklet, you have to fill in some particulars in the OMR Answer Sheet as per the instructions in your Admit Card.

7. On completion of the Examination, you should hand over the **original copy of OMR Answer Sheet** issued to you to the Invigilator before leaving the Examination Hall. You are allowed to take with you the candidate's copy (second copy) of the OMR Answer Sheet along with the Test Booklet for your reference.

Candidate's full signature | **Invigilator's signature**

P.T.O.

Notation

$\mathbb{N}$	set of all natural numbers $\{1, 2, 3, \cdots\}$.
$\mathbb{Z}$	set of all integers $\{0, \pm1, \pm2, \pm3, \cdots\}$.
$\mathbb{Q}$	set of all rational numbers.
$\mathbb{R}$	set of all real numbers.
$\mathbb{C}$	set of all complex numbers.
$\mathbb{R}^n$	n-dimensional Euclidean space $\{(x_1, x_2, \cdots, x_n) : x_k \in \mathbb{R}, 1 \leq k \leq n\}$.
S_n	group of all permutations on n distinct symbols under composition of mappings.
$\mathbb{Z}_n$	group of congruence classes of integers modulo n.

1. Which one of the following sets is countable ?

(A) The set of all real numbers.
(B) The set of all algebraic numbers.
(C) The set of all transcendental numbers.
(D) The set of all irrational numbers.

2. Consider the following statements with regard to the metrics d_1 and d_2 on a non-empty set X.

I. $d = \max\{d_1, d_2\}$ is a metric on X.

II. $\rho = \min\{d_1, d_2\}$ is a metric on X.

Which one of the following options is correct ?

(A) Only I is true.
(B) Only II is true.
(C) Both I and II are true.
(D) Neither I nor II is true.

3. Consider $S = \{q \in \mathbb{Q} : q^2 \geq 2\}$ as a subset of the metric space $(\mathbb{Q}, d)$, where $d(x, y) = |x - y|$ $(x, y \in \mathbb{Q})$. Then S is ______.

(A) a closed, but not an open in $\mathbb{Q}$.
(B) neither an open nor a closed set in $\mathbb{Q}$.
(C) an open, but not a closed in $\mathbb{Q}$.
(D) both an open and a closed in $\mathbb{Q}$.

4. On the set $X=\{a, b, c\}$, which of the following is *not* a topology on X?

(A) $T_1=\{\emptyset, X\}$.
(B) $T_2=\{\emptyset, \{a\}, X\}$.
(C) $T_3=\{\emptyset, \{a\}, \{a, b\}, X\}$.
(D) $T_4=\{\emptyset, \{a\}, \{a, b\}, \{b, c\}, X\}$.

5. Which one of the following statements is *true* in a topological space ?

(A) Complement of a connected set is connected.

(B) If the interior of a set S is connected, then S is connected.

(C) If a set S is connected, then its closure $\overline{S}$ is connected.

(D) If the closure $\overline{S}$ of a set S is connected, then S is connected.

6. The proof of the sequence $\{x_n\}_{n\geq 1}$ defined by $x_n = \frac{1}{n}$ converges to 0 (zero) relies on

(A) only the Archimedian property of $\mathbb{R}$.

(B) only the completeness property of $\mathbb{R}$.

(C) both the completeness and the Archimedian property of $\mathbb{R}$.

(D) Neither the completeness nor the Archimedian property of $\mathbb{R}$.

7. For the sequence $\{x_n\}_{n\geq 1}$ defined by $x_n = n^2 \cos\left(\frac{2}{n^2} + \frac{\pi}{2}\right)$, which one of the following is *true* ?

(A) The sequence oscillates.

(B) The sequence diverges.

(C) -2 is the limit of the sequence.

(D) 1 is the limit of the sequence.

8. Consider the following statements.

I. $\sum_{n=1}^{\infty} \cos\left(\frac{1}{n^2}\right)$ is a convergent series.

II. $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n^2}\right)$ is a convergent series.

Which one of the following options is *true* ?

(A) Both I and II are true.

(B) I is true, but II is false.

(C) I is false, but II is true.

(D) Neither I nor II is true.

9. Consider the following statements and pick out the correct option.

I. The sequence of functions $\{f_n\}_{n\geq 1}$ defined by $f_n(x) = \frac{nx}{1+(nx)^2}$ is uniformly convergent on $[0, 1]$.

II. The sequence of functions $\{f_n\}_{n\geq 1}$ defined by $f_n(x) = \frac{x}{1+nx^2}$ is uniformly convergent on $[0, 1]$.

(A) Only I is true.

(B) Only II is true.

(C) Both I and II are true.

(D) Neither I nor II is true.

10. The function $f(x) = x - [x]$ $(x \in \mathbb{R})$ ($[x]$ denotes the greatest integer $\leq x$) is discontinuous at ________

(A) all integers.

(B) all integers except 0 and 1.

(C) all integers except 1.

(D) all integers except 0.

11. If the function $f(x) = a\,|\sin(x)| + be^{|x|} + c|x|^3$ is differentiable at $x = 0$, then which one of the following is *true* ?

(A) $a = b = c = 0$.

(B) $a = 0, b = 0$ and $c \in \mathbb{R}$.

(C) $b = 0, c = 0$ and $a \in \mathbb{R}$.

(D) $a = 0, c = 0$ and $b \in \mathbb{R}$.

12. If f is a real valued function defined by $f(t) = \int_{x^2}^{4x^2} \cos\sqrt{t}\,dt$, then $f'\left(\frac{\pi}{2}\right) =$ ________.

(A) -4π

(B) $-4\pi - 1$

(C) $-\pi$

(D) 0

13. Which one of the following is *true* ?

(A) If $f : [a, b] \longrightarrow \mathbb{R}$ is a bounded function, then f is Riemann integrable.

(B) If f is Riemann integrable on $[a, b]$, then f is continuous on $[a, b]$.

(C) Let $f : [a, b] \longrightarrow \mathbb{R}$ be a real function. If $|f|$ is Riemann integrable, then f is Riemann integrable on $[a, b]$.

(D) If $f : [a, b] \longrightarrow \mathbb{R}$ is a continuous function except at a finite number of points of $[a, b]$, then f is Riemann integrable.

14. Which one of the following represents the value of $\lim_{n\to\infty} \sum_{k=1}^{2n} \frac{1}{n+k}$ in terms of the Riemann integral ?

(A) $\int_0^1 \frac{dx}{1+x}$.

(B) $\int_0^2 \frac{dx}{1+x}$.

(C) $\int_0^3 \frac{dx}{1+x}$.

(D) $\int_1^3 \frac{dx}{1+x}$.

15. The maximum and minimum value of the function $f(x, y) = 5x^2 + 2xy + 5y^2$ on the circle $x^2 + y^2 = 1$ denoted by $\max(f)$ and $\min(f)$, respectively are ________.

(A) $\max(f) = 6, \min(f) = 0$.

(B) $\max(f) = 6, \min(f) = 4$.

(C) $\max(f) = \min(f) = 5$.

(D) $\max(f) = \infty, \min(f) = -\infty$.

16. The parabolic arc $y = \sqrt{x}, 1 \le x \le 2$ is revolved around the X-axis. The volume of the solid of revolution is ________.

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{2}$

(C) $\frac{3\pi}{4}$

(D) $\frac{3\pi}{2}$

17. Stoke's theorem is used to convert ________.

(A) surface integral into volume integral.

(B) line integral into volume integral.

(C) line integral into surface integral.

(D) volume integral into line integral.

18. Using Green's theorem, the value of the integral: $\int_C (xy\, dy - y^2\, dx)$, where C is the square cut from the first quadrant by the lines $x = 1$ and $y = 1$ is ________.

(A) $\frac{5}{3}$ (B) $\frac{3}{2}$ (C) 1 (D) $\frac{1}{2}$

19. Which one of the following statements is *false* ?

(A) The outer measure of any countable set of $\mathbb{R}$ is same as the cardinality of the set.

(B) If $X = \{a, b, c, d\}$, then $\sum = \{\emptyset, \{a, b\}, \{c, d\}, \{a, b, c, d\}\}$ is a sigma algebra on X.

(C) The characteristic function χ_S is measurable, if and only if S is a measurable set.

(D) Every open set is a Borel set.

20. Which one of the following is *true* ?

(A) The Cantor set C is an open, uncountable set of Lebesgue measure zero.

(B) The Cantor set C is a closed, uncountable set of Lebesgue measure zero.

(C) The Cantor set C is an open, countable set of Lebesgue measure zero.

(D) The Cantor set C is a closed, countable set of Lebesgue measure zero.

21. Let $S = [0, 1] \cap \mathbb{Q}$. Consider the function $f : [0, 1] \longrightarrow \mathbb{R}$ defined as $f(x) = \chi_S(x)$ (χ_S is the characteristic function of S). Then, which one of the following is *true* ?

(A) f is Riemann integrable as well as Lebesgue integrable.

(B) f is Riemann integrable, but not Lebesgue integrable.

(C) f is Lebesgue integrable, but not Riemann integrable.

(D) f is neither Riemann integrable, nor Lebesgue integrable.

22. If $\{f_n\}_{n\geq 1}$ is a sequence of non-negative measurable functions on a measure space $(X, \mathcal{M}, \mu)$, then which one of the following is *true* ?

(A) $\int_X \liminf_{n\to\infty} f_n\, d\mu \leq \liminf_{n\to\infty} \int_X f_n\, d\mu$.

(B) $\int_X \liminf_{n\to\infty} f_n\, d\mu \geq \liminf_{n\to\infty} \int_X f_n\, d\mu$.

(C) $\int_X \liminf_{n\to\infty} f_n\, d\mu = \liminf_{n\to\infty} \int_X f_n\, d\mu$.

(D) $\int_X \liminf_{n\to\infty} f_n\, d\mu \neq \liminf_{n\to\infty} \int_X f_n\, d\mu$.

23. For $a, b, c \in \mathbb{R}$, if the differential equation: $(ax^2 + bxy + y^2)dx + (2x^2 + cxy + y^2)dy = 0$ is exact, then

(A) $b = 2, c = 2a$. (B) $b = 4, c = 2$. (C) $b = 2, c = 4$. (D) $b = 2, a = 2c$.

24. The particular integral of $(D^2 + 1)y = \cos(x)$ $(D = \frac{d}{dx})$ is ________.

(A) $\frac{x\sin(x)}{2}$. (B) $\frac{\sin(x)}{2}$. (C) $-\frac{x\sin(x)}{2}$. (D) $-\frac{\sin(x)}{2}$.

25. Which one of the following is the solution of the initial value problem: $\frac{dy}{dx} = 1 + y^2$, $y(0) = 1$? ....

(A) $y(x) = \text{cosec}\left(x + \frac{\pi}{4}\right)$.

(B) $y(x) = \sec\left(x + \frac{\pi}{4}\right)$.

(C) $y(x) = \tan\left(x + \frac{\pi}{4}\right)$.

(D) $y(x) = \cot\left(x + \frac{\pi}{4}\right)$.

26. Consider the following statements regarding the two solutions $y_1(x) = \sin(x)$ and $y_2(x) = \cos(x)$ of the differential equation: $y'' + y = 0$.

I. They are linearly dependent solutions of the differential equation.

II. Their wronskian is 1.

III. They are linearly independent solutions of the differential equation.

Then, choose the correct option.

(A) Only I is true.

(B) Only III is true.

(C) Only I and II are true.

(D) Only II and III are true.

27. How many solution(s) does the initial value problem: $\frac{dy}{dx} = \frac{2y}{x}$, $y(0) = 0$ have ?

(A) No solution.

(B) Unique solution.

(C) Two solutions.

(D) Infinitely many solutions.

28. Let $P_n(x)$ denote the Legendre polynomial of degree n. Then, the value of $\int_{-1}^{1} x^{99} P_{100}(x)\,dx$ is ________.

(A) -1

(B) 0

(C) $\frac{99}{100}$

(D) 1

29. If $J_n(x)$ denotes the Bessel's function of first kind, then which one of the following is *true* ?

(A) $J_2(x) = 2J_1(x) - xJ_0(x)$.

(B) $J_2(x) = \frac{4}{x}J_1(x) - J_0(x)$.

(C) $J_2(x) = 2J_1(x) - \frac{2}{x}J_0(x)$.

(D) $J_2(x) = \frac{2}{x}J_1(x) - J_0(x)$.

30. If δ denotes the Dirac Delta function, then which one of the following is the Laplace transform of $3\delta(t-4)$?

(A) e^{-4s}.

(B) e^{4s}.

(C) $3e^{-4s}$.

(D) $3e^{4s}$.

31. The Fourier series expansion of the function $f(x) = x^3$ in the interval $[-1, 1)$ with periodic continuation has ________.

(A) only cosine terms.

(B) only sine terms.

(C) both sine and cosine terms.

(D) only sine terms with a non-zero constant.

32. The solution of the partial differential equation: $\frac{\partial u}{\partial t} + 3\frac{\partial u}{\partial x} + 2u(x,t) = 0$ with $u(x, 0) = \sin x^2$ is ________.

(A) $u(x,t) = \sin(x^2 - 4t^2)$.

(B) $u(x,t) = e^{-2t}\sin(x - 3t)^2$.

(C) $u(x,t) = e^{-2t}\sin(x - 2t)^2$.

(D) $u(x,t) = e^{-2t}\sin(x^2)$.

33. Let V be the set consisting of all real functions $y = f(x)$ satisfying $\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = 0$. Then which one of the following is *true* ?

(A) V is not a real vector space.

(B) V is a 1-dimensional real vector space.

(C) V is 2-dimensional real vector space.

(D) V is 3-dimensional real vector space.

34. On the vector space V of all 2×2 real matrices with respect to usual matrix addition and multiplication over the field $\mathbb{R}$, consider the following statements.

I. W_1, a subset of V consisting of all matrices with zero determinant is a subspace of V.

II. W_2, a subset of V consisting of all matrices A such that $A^2 = A$ is a subspace of V.

Then, which one of the following options is *true* ?

(A) Only I is true.

(B) Only II is true.

(C) Neither I nor II is true.

(D) Both I and II are true.

35. Let V be a vector space of dimension $n < \infty$. Then, which one of the following statements is *false* ?

(A) Every basis of V has the same number of elements.

(B) Any subset of V containing more than n vectors is linearly independent.

(C) Any linearly independent set is a part of a basis for V.

(D) A linearly independent set with n elements is a basis for V.

36. Let U and W be distinct 4-dimensional subspaces of a vector space V of dimension 6. Then, the possible dimension(s) of the subspace $U \cap W$ is/are ________.

(A) 2

(B) 1 or 2

(C) 2 or 3

(D) 3 or 4

37. If T is a linear transformation defined on $\mathbb{R}^3$ by $T(x, y, z) = (3x, x - y, 2x + y + z)$ for all $(x, y, z) \in \mathbb{R}^3$, then which one of the following is *true* ?

(A) Rank(T)=4. (B) Rank(T)=3. (C) Rank(T)=2. (D) Rank(T)=1.

38. The eigenvalues of the matrix $M = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ are ________.

(A) $0, 1, 2$ (B) $-1, 0, 1$ (C) $2, 1 - i, 1 + i$ (D) $-1, -1, 2$

39. If S and T are 3×3 real matrices such that $\text{rank}(ST) = 1$, then which one of the following *cannot* be the rank of TS ?

(A) 3 (B) 2 (C) 1 (D) 0

40. Which one of the following is *true* ?

(A) The matrix $S = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ and the matrix $T = \begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix}$ are similar.

(B) The matrix $S = \begin{bmatrix} -1 & 6 \\ -2 & 6 \end{bmatrix}$ and the matrix $T = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$ are similar.

(C) The matrix $S = \begin{bmatrix} 0 & 1 \\ 5 & 3 \end{bmatrix}$ and the matrix $T = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$ are similar.

(D) The matrix $S = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and the matrix $T = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ are similar.

41. Let $(G, \star)$ be an algebraic structure, where $G = \mathbb{R} - \{0\}$. If the binary operation '$\star$' on G is defined by $a \star b = \frac{ab}{4}$ for all $a, b \in G$, then the inverse of a in G is________.

(A) $\frac{a}{4}$. (B) $16a$. (C) $\frac{16}{a}$. (D) $\frac{4}{a}$

42. Which one of the following is *false* ?

(A) A group of order 6 is cyclic. (B) A group of order 11 is cyclic.

(C) A group of order 15 is cyclic. (D) A group of order 35 is cyclic.

43. Which one of the following is *false* ?

(A) Number of distinct elements in S_4 is 24. (B) Number of 2-cycles in S_4 is 6.

(C) Number of 3-cycles in S_4 is 6. (D) Number of 4-cycles in S_4 is 6.

44. Consider the following statements and pick out the correct option.

I. There exists an infinite group with all elements of finite order.

II. In a group G, if $x \in G$ with $x^9 = e$ and $x^{11} = e$, then $x = e$, where e is the identity element of G

(A) Both I and II are false.
(B) I is true, but II is false.
(C) I is false, but II is true.
(D) Both I and II are true.

45. The set of all ring homomorphisms $\phi : \mathbb{Z}_{12} \longrightarrow \mathbb{Z}_{28}$

(A) has four elements. (B) has two elements. (C) is a singleton set. (D) is an empty set.

46. For the ideal $I =< x^2 + 1 >$ of $\mathbb{Z}[x]$, which one of the following is *true* ?

(A) I is a prime ideal, but not a maximal ideal.

(B) I is a maximal ideal, but not a prime ideal.

(C) I is neither a prime ideal nor a maximal ideal.

(D) I is both a prime and a maximal ideal.

47. Which one of the following rings is *not* a field ?

(A) $\dfrac{\mathbb{R}[x]}{< x^2 + 1 >}$.
(B) $\dfrac{\mathbb{Q}[x]}{< x^2 + 1 >}$.
(C) $\dfrac{\mathbb{Z}_2[x]}{< x^2 + 1 >}$.
(D) $\dfrac{\mathbb{Z}_3[x]}{< x^2 + 1 >}$.

48. The number of Sylow 3-subgroups of S_4 is ________.

(A) 5
(B) 4
(C) 3
(D) 2

49. Let $\mathbb{Q}$ denote the field of rational numbers. Then the degree of the splitting field of $x^3 - 2$ over $\mathbb{Q}$ is ________.

(A) 2
(B) 3
(C) 5
(D) 6

50. If $27! \equiv x \pmod{29}$, then the value of x is ________.

(A) 1
(B) 5
(C) 13
(D) 27

51. For the function $f(z) = x^2 + y^2 + 2ixy$, $(z = x + iy \in \mathbb{C})$, which one of the following is *true* ?

(A) f is differentiable only at $z = 0$.

(B) f is differentiable only at the points that lie on the X-axis.

(C) f is differentiable only at the points that lie on the Y-axis.

(D) f is differentiable only at the points that lie on both the co-ordinate axes.

52. Let $f = u + iv$ be a non-constant analytic function in a domain G of the complex plane $\mathbb{C}$. For $z = x + iy \in G$, consider the following statements.

I. $g(z) = u_x(x, y) - iu_y(x, y)$ is analytic in G.

II. $h(z) = v_x(x, y) + iv_y(x, y)$ is analytic in G.

Then, which one of the following options is *true* ?

(A) I is true, but II is false. (B) I is false, but II is true.

(C) Both I and II are true. (D) Neither I nor II is true.

53. The radius of convergence of the power series: $\sum_{n=0}^{\infty}\left(1+\frac{1}{n}\right)^{n\,(n!)^2} z^{(n!)^2}$ is ______.

(A) 1 (B) e (C) $\frac{1}{e}$ (D) $\frac{1}{e^2}$

54. Which one of the following is the image of the unit circle $|z| = 1$ under the Bilinear transformation $w = u + iv = \frac{1}{z}$?

(A) $4u + 1 = 0$. (B) $2u - 1 = 0$. (C) $2v + 1 = 0$. (D) $u + v > 2$.

55. The value of the integral: $\int_C \bar{z}\, dz$, where C is the upper half of the circle $|z| = 2$ from $z = -2i$ to $z = 2i$ is equal to ______.

(A) $-4i\pi$ (B) $-2i\pi$ (C) $2i\pi$ (D) $4i\pi$

56. Consider the following statements and choose the correct option.

I. $|\sin(z)| \leq 1$ for all $z \in \mathbb{C}$.

II. If f is an entire function satisfying $|f(z)| \leq 2\log(1+|z|)$ for all $z \in \mathbb{C}$, then $f(z) = 0$ for all $z \in \mathbb{C}$.

(A) Only I is true. (B) Only II is true.

(C) Both I and II are true. (D) Neither I nor II is true.

57. The value of the integral $\int_{|z|=2} \frac{\cos(z)\, dz}{z^3}$ is ______.

(A) $i\pi$ (B) $2i\pi$ (C) $-2i\pi$ (D) $-i\pi$

58. In the Laurent series expansion of $f(z) = \frac{1}{(z-1)(z-2)}$ in the region $0 < |z-1| < 1$, the coefficient of $(z-1)^{-1}$ is ______.

(A) 1 (B) 0 (C) -1 (D) -2

59. If $f(z) = \mathrm{Log}(z)$, then $z = 0$ is ________.

(A) a removal singularity of f.
(B) a pole of f.
(C) an essential singularity of f.
(D) a non-isolated singularity of f.

60. The residue of the function $f(z) = \frac{e^z}{z}$ at ∞ is ________.

(A) $-\frac{1}{2}$
(B) $\frac{1}{2}$
(C) -1
(D) 1

61. The next iterative value of the root of $x^2 - 4 = 0$ using secant method, if the initial guesses are 3 and 4 is ________.

(A) 2.2857
(B) 2.5000
(C) 5.5000
(D) 5.7143

62. Using the following values of x and $f(x)$,

x	0	0.5	1	1.5
$f(x)$	1	α	0	$-\frac{5}{4}$

if the integral $\int_0^{1.5} f(t)\,dt$, evaluated by Trapezoidal rule is $\frac{5}{16}$, then the value of α is ________.

(A) $\frac{19}{24}$
(B) $\frac{7}{3}$
(C) $\frac{5}{4}$
(D) $\frac{3}{4}$

63. Exact value of the definite integral: $\int_a^b f(x)dx$ by using Simpson's rule

(A) is obtained when f is a polynomial of degree 3.
(B) is obtained when f is a polynomial of degree 4.
(C) is obtained when f is a polynomial of degree 5.
(D) cannot be obtained for any polynomial.

64. The integral $\int_5^{10} f(x)\,dx$ is exactly same as which one of the following ?

(A) $\int_{-1}^{1} f(2.5x + 7.5)\,dx$.
(B) $2.5\int_{-1}^{1} f(2.5x + 7.5)\,dx$.
(C) $5\int_{-1}^{1} f(5x + 5)\,dx$.
(D) $5\int_{-1}^{1} f(2.5x + 7.5)\,dx$.

65. Consider the following $x - y$ data.

x	15	18	22
y	24	37	25

If the Newton's divided difference second order polynomial for the above data is given by $f_2(x) = b_0 + b_1(x-15) + b_2(x-15)(x-18)$, then the value of b_1 is ________.

(A) $-\frac{21}{20}$ (B) $\frac{3}{20}$ (C) $\frac{13}{3}$ (D) $\frac{22}{3}$

66. If the quadrature formula $\int_0^2 xf(x)\,dx \approx af(0) + bf(1) + cf(2)$ is exact for all polynomials of degree ≤ 2, then the value of $2b - c$ is ________.

(A) $\frac{8}{3}$. (B) 2. (C) $\frac{4}{3}$. (D) $\frac{2}{3}$.

67. The central difference operator δ and backward difference operator ∇ are related as

(A) $\delta = \nabla(1+\nabla)^{\frac{1}{2}}$. (B) $\delta = \nabla(1+\nabla)^{-\frac{1}{2}}$. (C) $\delta = \nabla(1-\nabla)^{\frac{1}{2}}$. (D) $\delta = \nabla(1-\nabla)^{-\frac{1}{2}}$.

68. The Newton-Raphson formula for finding the cube root of N is ________.

(A) $\frac{2x_n^3 + N}{3x_n^2}$. (B) $\frac{2x_n^3 - N}{3x_n^2}$. (C) $\frac{2x_n^3 + N^2}{3x_n^2}$. (D) $\frac{2x_n^3 - N^2}{3x_n^2}$.

69. Given the following data with the interpolating polynomial p

x	2	5
$p(x)$	1.5	4

the value of $p(3)$ is ________.

(A) $\frac{11}{6}$ (B) $\frac{7}{3}$. (C) $\frac{5}{3}$ (D) $\frac{3}{2}$

70. Using the mid-point method, the approximate value of the differential equation: $y' = y^2 + t^2, y(0) = 1$ is ________.

(A) 0.00025 (B) 0.0025 (C) 0.025 (D) 0.25

71. Which one of the following is *not* a norm on $\mathbb{R}^2$?

(A) $\|(x,y)\| = |x| + |y|,\ (x,y) \in \mathbb{R}^2$.

(B) $\|(x,y)\| = \left(|x|^{\frac{1}{2}} + |y|^{\frac{1}{2}}\right)^2,\ (x,y) \in \mathbb{R}^2$.

(C) $\|(x,y)\| = (|x|^2 + |y|^2)^{\frac{1}{2}},\ (x,y) \in \mathbb{R}^2$.

(D) $\|(x,y)\| = (|x|^3 + |y|^3)^{\frac{1}{3}},\ (x,y) \in \mathbb{R}^2$.

72. Let $X = C([0,1])$ be the vector space of all continuous functions on $[0,1]$. For $f \in X$, let $\|f\|_1 = \int_0^1 |f(t)|\,dt$ and $\|f\|_\infty = \sup_{0\leq t\leq 1} |f(t)|$ be the norms on X. Then, which one of the following is true ?

(A) Both $(X, \|\cdot\|_1)$ and $(X, \|\cdot\|_\infty)$ are Banach spaces.

(B) $(X, \|\cdot\|_1)$ is a Banach space, but $(X, \|\cdot\|_\infty)$ is not a Banach space.

(C) $(X, \|\cdot\|_1)$ is not a Banach space, but $(X, \|\cdot\|_\infty)$ is a Banach space.

(D) Neither $(X, \|\cdot\|_1)$ nor $(X, \|\cdot\|_\infty)$ are Banach spaces.

73. Let $T : \ell^2 \longrightarrow \ell^2$ be the linear operator defined by $T(x) = (0, x_1, x_2, \ldots)$, for all $x = (x_1, x_2, \ldots) \in \ell^2$. Then, which one of the following is *true* ?

(A) $\|T\| = \frac{1}{\sqrt{2}}$. (B) $\|T\| = 1$. (C) $\|T\| = \sqrt{2}$. (D) $\|T\| = \infty$.

74. Which of the following statements is *correct* ?

(A) A finite dimensional vector space is a Banach space with respect to any norm on it.

(B) On every vector space X over $\mathbb{R}$ or $\mathbb{C}$, there is a norm with respect to which X is a Banach space.

(C) If $(X, \|\cdot\|)$ is a normed space and Y is a subspace of X, then every bounded linear functional f on Y has a unique bounded linear extension g to X such that $\|g\| = \|f\|$.

(D) The dual of a separable Banach space is separable.

75. If X, Y are normed spaces and $T : X \longrightarrow Y$ is a surjective continuous linear map, then which one of the following is *true* ?

(A) T is always an open map.

(B) T is an open map, if X is a Banach space.

(C) T is an open map, if Y is a Banach space.

(D) T is an open map, if both X and Y are Banach spaces.

76. Which one of the following is *false* ?

(A) $\mathbb{R}^n$ is reflexive. (B) $C([0,1])$ is reflexive.

(C) ℓ^p $(p > 1)$ is reflexive. (D) $L^p([0,1])$ $(p > 1)$ is reflexive.

77. On a normed linear space X, which one of the following is *false* ?

(A) If a sequence $\{x_n\}_{n\geq 1}$ is convergent to x in X, then $\{x_n\}_{n\geq 1}$ is weakly convergent to x.

(B) If a sequence $\{x_n\}_{n\geq 1}$ is weakly convergent to x in X, then $\{x_n\}_{n\geq 1}$ is convergent to x, if the dimension of X is finite.

(C) If a sequence $\{f_n\}_{n\geq 1}$ is weakly convergent to f in X' (dual space of X), then $\{f_n\}_{n\geq 1}$ is weak* convergent to f.

(D) If a sequence $\{f_n\}_{n\geq 1}$ is weak* convergent to f in X' (dual space of X), then $\{f_n\}_{n\geq 1}$ is convergent to f.

78. In an inner product space $(V, <\cdot, \cdot>)$, which one of the following statements is *true* ?

(A) Every orthonormal set in V must be a basis for V.

(B) Every orthonormal set in V must span V, but need not necessarily be linearly independent.

(C) Every orthonormal set in V must be linearly independent in V, but need not necessarily be a basis for V.

(D) Every orthonormal set in V must be finite.

79. If $\{x, y\}$ is an orthonormal set in an inner product space $(V, <\cdot, \cdot>)$, then what is the value of $\|x - y\| + \|x + y\|$?

(A) $\sqrt{2}$. (B) 2. (C) $2\sqrt{2}$. (D) $2 + \sqrt{2}$.

80. The sequence space ℓ^p $(p \geq 1)$ is a Hilbert space, if and only if

(A) $p > 1$. (B) $p = 2$.

(C) p is an even integer. (D) $p = \infty$.

81. The premises: $(p \wedge q) \vee r$ and $r \longrightarrow s$ imply which one of the following conclusion ?

(A) $p \vee s$ (B) $p \vee r$ (C) $p \vee q$ (D) $q \vee r$

82. Parul is out for a trip or it is not snowing and it is snowing or Raju is playing chess imply which one of the following ?

(A) Parul is out for a trip.

(B) Raju is playing chess.

(C) Parul is out for a trip and Raju is playing chess.

(D) Parul is out for a trip or Raju is playing chess.

83. Consider the following statements for a simple graph G.

I. The adjacency matrix of G is symmetric.

II. Trace of the adjacency matrix of G is 1.

Which one of the following options is *true* ?

(A) Only I. (B) Only II. (C) Both I and II. (D) Neither I nor II.

84. Which one of the following is *false* ?

(A) $(\{1, 2, 3, 6, 9, 18\}, |)$ is a bounded lattice. (B) $(\mathbb{Z}, \leq)$ is not a bounded lattice.

(C) $((0, 1), \leq)$ is a bounded lattice. (D) $([0, 1], \leq)$ is a bounded lattice.

85. In the poset $(\mathbb{Z}^+, |)$ ($\mathbb{Z}^+$ is the set of all positive integers and "|" is the divides relation), than the integers 9 and 351 are ________.

(A) comparable. (B) not comparable.

(C) comparable, but not determined. (D) determined, but not comparable.

86. Which one of the following is the solution of the recurrence relation:
$a_n = 6a_{n-1} - 9a_{n-2}$, $a_0 = 1, a_1 = 6$?

(A) $a_n = (n+1)6^n$. (B) $a_n = (n+1)3^n$. (C) $a_n = (n+1)5^n$. (D) $a_n = (n-1)3^n$.

87. Which one of the following is the minimized form of the Boolean expression:
$F(x, y, z) = \overline{x}\,\overline{z} + \overline{y}\,\overline{z} + y\overline{z} + xyz$?

(A) $\overline{x}\,\overline{y} + z$. (B) $\overline{x}y + z$. (C) $\overline{z} + xy$. (D) xyz.

88. Suppose a complete binary tree has height $h > 0$. The minimum no of leaf nodes possible in term of h is ________.

(A) $2^h - 1$ (B) $2^{h-1} + 1$ (C) 2^{h-1} (D) $2^h + 1$

89. Let $\star$ be the binary operation defined by $x \star y = x' + y'$, where x and y are Boolean variables. Assuming that x, y and z are Boolean variables, consider the following statements.

I. $(x \star y) \star z = x \star (y \star z)$.

II. $y \star z = z \star y$.

Which one of the following options is *true* ?

(A) Neither I nor II. (B) Both I and II. (C) Only I. (D) Only II.

90. The graph in which, there is a closed trail which includes every edge of the graph is known as ________.

(A) Hamiltonian Graphs. (B) Euler Graphs.

(C) Planar graph. (D) Directed Graph.

91. Consider the following Linear Programming Problem (LPP):

Maximize: $Z = 7x_1 + 6x_2 + 4x_3$

subject to: $x_1 + x_2 + x_3 \leq 5$, $2x_1 + x_2 + 3x_3 \leq 10$; $x_1, x_2, x_3 \geq 0$.

The number of basic solution(s) is ________.

(A) 10 (B) 9 (C) 8 (D) 7

92. The feasible region represented by the constraints: $x_1 - x_2 \leq 1, x_1 + x_2 \geq 3; x_1, x_2 \geq 0$ of the objective function: Maximize $Z = 3x_1 + 2x_2$ is ______

(A) a polygon. (B) an unbounded feasible region.

(C) a point. (D) an empty set.

93. The variable that is included in the "$\leq$" type inequality constraint for the purpose of converting a general form of LPP to standard form of LPP is called as ______.

(A) surplus variable (B) artificial variable (C) basic variable (D) slack variable

94. In a LPP, suppose there are 3 basic variables and 2 non-basic variables, then the possible number of basic solutions are ______.

(A) 6 (B) 8 (C) 10 (D) 12

95. Which one of the following methods is commonly used to solve assignment problems ?

(A) Stepping stone method. (B) North-West corner method.

(C) Hungarian method. (D) Vogel's approximation method.

96. Consider the following statements with respect to a LPP:

I. The dual of the dual linear programming problem is again the primal problem.

II. If either the primal or the dual problem has an unbounded objective function value, the other problem has no feasible solution.

III. If either the primal or the dual problem has a finite optimal solution, the other one also possess the same, and the optimal value of the objective functions of the two problems are equal.

Which of the following options is *true* ?

(A) I and II only. (B) I and III only. (C) II and III only. (D) I, II and III.

97. In the optimal simplex table, what does $z_j - c_j = 0$ value indicates ?

(A) an alternative solution. (B) cycling.

(C) an infeasible solution. (D) an unbounded solution.

98. Which one of the following is the objective function of the dual of the following LPP:

$$\text{Minimize } Z = 15x_1 + 12x_2$$

subject to the constraints: $x_1 + 2x_2 \leq 3, 2x_1 - 4x_2 \leq 5;\ x_1, x_2 \geq 0.$

(A) Maximize $Z = y_1 + y_2$. (B) Maximize $Z = 3y_1 + 5y_2$.

(C) Maximize $Z = y_1 + 2y_2$ (D) Maximize $Z = 2y_1 - 4y_2$.

99. In game theory, a situation in which one firm can gain only what another firm loses is called ______

(A) a zero-sum game.

(B) a non-zero-sum game.

(C) a prisoners' dilemma.

(D) a predation game.

100. What is the traveling salesman problem equivalent to in graph theory ?

(A) Any circuit.

(B) A Hamilton circuit in a non-weighted graph.

(C) A round trip airfare.

(D) A Hamilton circuit in a weighted graph.

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