DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO

TEST BOOKLET

Sl. No. 01811

Subject Code : 18

Subject : Mathematics

LECTURERS FOR NON-GOVT. AIDED COLLEGES OF ODISHA

Time Allowed : 3 Hours

Maximum Marks : 165

: INSTRUCTIONS TO CANDIDATES :

1. **IMMEDIATELY AFTER THE COMMENCEMENT OF THE EXAMINATION, YOU SHOULD CHECK THAT THIS TEST BOOKLET CONTAINS 31 PAGES AND DOES NOT HAVE ANY UNPRINTED OR TORN OR MISSING PAGES OR ITEMS ETC. IF SO, GET IT REPLACED BY A COMPLETE TEST BOOKLET.**

2. You have to enter your **Roll No.** on the Test Booklet in the Box provided alongside. **DO NOT** write anything else on the Test Booklet.

3. The Test Booklet contains **165** questions. Each question comprises four answers. You have to select the correct answer which you want to mark (darken) on the Answer Sheet. In case, you feel that there is more than one correct answer, you should mark (darken) the answer which you consider the best. In any case choose ONLY ONE answer for each question. If more than one answer is darkened it will be considered as wrong.

4. You have to mark (darken) all your answers **ONLY** on the **separate OMR Answer Sheet** provided, by **using BLACK BALL POINT PEN**. You have to do rough work on the space provided in the Test Booklet only. See instruction in the Answer Sheet.

5. All questions carry equal marks, i.e. of one mark for each correct answer and each wrong answer will result in negative marking of **0.25** mark.

6. Before you proceed to mark (darken) in the Answer Sheet the answers to various questions in the Test Booklet, you have to fill in some particulars in the Answer Sheet as per the instructions in your Admit Card.

7. After you have completed filling in all your answers on the Answer Sheet and after completion of the examination, you should hand over to the Invigilator the **Original Answer Sheet (OMR Answer Sheet)** issued to you. You are allowed to take with you the candidate's copy/second page of the Answer Sheet along with the Test Booklet after completion of the examination for your reference.

Candidate's full signature

Invigilator's signature

2018

SEAL

1. Which of the following is not correct ?

(A) The set $\mathbb{Q}^+$ of positive rationals is a group under ordinary multiplication.

(B) The subset {1, – 1, i, –i} of complex numbers is a group under complex multiplication.

(C) The set $\mathbb{Z}_n = \{0, 1, 2, ...n-1\}$ for $n \geq 1$ is a group under addition modulo n.

(D) The set {0, 1, 2, 3} is a group under multiplication modulo 4.

2. The set $\mathbb{Z}_n = \{0, 1, 2, ..., n-1\}$ is a group under multiplication modulo n if and only if :

(A) n is a prime

(B) n is even

(C) n is odd

(D) n is not a prime

3. Let G be a group and let a be an element of order n in G. If $a^k = e$, then :

(A) n divides k

(B) k divides n

(C) n does not divide k

(D) n and k are primes

4. The order of (123)(145) in the permutation group S_5 is :

(A) 6

(B) 3

(C) 5

(D) 9

5. The group of even permutations of n symbols is denoted by A_n and it is called alternating group of degree n. For $n > 1$, A_n has order :

(A) n!

(B) $\frac{(n+1)!}{2}$

(C) $\frac{(n-1)!}{2}$

(D) $\frac{n!}{2}$

6. For every integer a and every prime p :

(A) $p^a \bmod p = p \bmod a$

(B) $a^p \bmod p = a \bmod p$

(C) $a^p \bmod p = p \bmod p$

(D) $a^p \bmod p = p \bmod a$

7. The group of rotations of a cube is isomorphic to :

(A) A_4

(B) S_5

(C) S_4

(D) A_5

8. An integral domain is a commutative ring with unity and :

(A) Zero-divisors

(B) No zero-divisors

(C) Zero multipliers

(D) None of these

9. The characteristic of an integral domain is :

(A) 0 or non-prime

(B) 0 or even number

(C) 0 or odd number

(D) 0 or prime

10. If F is a field of characteristics 0, then F contains a subfield isomorphic to the :

(A) Irrational numbers

(B) Rational numbers

(C) Even numbers

(D) Odd numbers

11. The polynomial $3x^5 + 15x^4 - 20x^3 + 10x + 20$ is irreducible over :

(A) $\mathbb{R}$

(B) $\mathbb{R} - \mathbb{Q}$

(C) $\mathbb{Q}$

(D) $\mathbb{Z}$

12. Let G be a group and $a, b \in G$ such that $o(a) = 6$, $o(b) = 2$ and $a^3b = ba$. Then $o(ab)$ is :

(A) 6

(B) 8

(C) 12

(D) 2

13. The number of subgroups of order 2 in the permutation group S_3 is :

(A) 1

(B) 3

(C) 12

(D) 2

14. If G is an abelian group, then the number of conjugacy classes equal to :

(A) $o(G)$

(B) 1

(C) $o(G) - o(\mathbb{Z}(G))$

(D) 2

15. Let G be an abelian group with the identity e. Which one of the following statement is true ?

(A) $H = \{x \in G : \text{order of } x \text{ is odd}\}$ is a subgroup of G

(B) $H = \{x \in G : \text{order of } x \text{ is even}\} \cup \{e\}$ is a subgroup of G

(C) Every subgroup of G is normal

(D) G is cyclic

16. In the ring $\mathbb{Z}_8[x]$, the element $4x^2 + 6x + 3$ is :

(A) A nilpotent

(B) A unit

(C) A idempotent

(D) A non-zero divisor

17. Suppose that $\phi : \mathbb{Z}_{20} \to \mathbb{Z}_{20}$ is an automorphisim such that $\phi(5) = 5$, the number of possibilities for $\phi(1)$ is :

(A) 4

(B) 1

(C) 5

(D) 20

18. Let S_3 be the permutation group of $\{1, 2, 3\}$. Then there exists a non-trivial group homomorphism $f : S_3 \to S_3$ such that :

(A) Kernel $f = \{(12), e\}$

(B) Kernel $f = \{(123), (132), e\}$

(C) Kernel $f = \{(123), (12)\}$

(D) None of these

19. Let A be a 5 × 5 real matrix. Suppose 0 is one of eigenvalues of A. Which of the following statement is true ?

(A) System $Ax = 0$ has unique solution

(B) System $Ax = C$ has unique solution for any C

(C) System $Ax = 0$ has a non-trivial solution

(D) None of these

20. Which of the following subset is a subspace of the vector space $\mathbb{R}^3$ over the field $\mathbb{R}$?

(A) $\{(u, v, w) \in \mathbb{R}^3 : 2u + 3v + 4w = 0\}$

(B) $\{(u, v, w) \in \mathbb{R}^3 : 2u + 3v + 4w = 1\}$

(C) $\{(u, v, w) \in \mathbb{R}^3 : u > 0, v > 0, w > 0\}$

(D) $\{(u, v, w) \in \mathbb{R}^3 : u, v, w \text{ are rationals}\}$

21. The dimension of the vector space $\mathbb{Q}[\sqrt{2}]$ over the field $\mathbb{Q}$ is:

(A) 1

(B) ∞

(C) 4

(D) 2

22. Let X and Y be subspaces of finite dimensional vector space V. Let $X + Y = \{x + y : x \in X, y \in Y\}$. The dimension of the subspace X + Y is always equal to :

(A) $\dim(X + Y) = \dim(X) + \dim(Y) - \dim(X \cap Y)$

(B) $\dim(X + Y) = \dim(X) + \dim(Y)$

(C) $\dim(X + Y) = \max\{\dim(X), \dim(Y)\}$

(D) $\dim(X + Y) = \dim(X) + \dim(Y) + \dim(X \cap Y)$

23. Suppose G is a finite group and H is a subgroup of G. If $[G : H] = 2$, then which of the following statement is true ?

(A) If $x \in H$ and $y \notin H$, then $xy \in H$

(B) If $x \notin H$ and $y \notin H$, then $xy^{-1} \in H$

(C) If $x \notin H$ and $y \notin H$, then $xy \in H$

(D) Both (B) and (C) are true

24. The characteristic polynomial of the 3×3 matrix $A = \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}$ is given by :

(A) $-\lambda^3 + (a + b + c)\lambda^2 - (ab + bc + ca)\lambda + abc$

(B) $\lambda^3 + (a + b + c)\lambda^2 - (ab + bc + ca)\lambda + abc$

(C) $-\lambda^3 - (a + b + c)\lambda^2 - (ab + bc + ca)\lambda + abc$

(D) $\lambda^3 + (a + b + c)\lambda^2 + (ab + bc + ca)\lambda + abc$

25. The greatest common divisor (gcd) of $5n + 3$ and $7n + 4$, for all $n \in \mathbb{N}$ is :

(A) 1

(B) 5

(C) n

(D) 2

26. How many three digit numbers are divisible by 6 ?

(A) 142

(B) 150

(C) 148

(D) 166

27. If gcd (1492, 1066) = 2, then we have ℓcm (1492, 1066) = ?

(A) 752936

(B) 795326

(C) 795236

(D) Can not say

28. Let a, b $\in \mathbb{Z}$ such that gcd (a, b) = 5. Then the equation $ax + by = c^2$ has :

(A) Unique solution for any $c \in \mathbb{Z}$

(B) Infinitely many solutions if $(25, c^2) = 1$

(C) No solution for any $c \in \mathbb{Z}$

(D) Infinitely many solutions $(x,y) \in \mathbb{Z} \times \mathbb{Z}$ if $(25, c^2) \neq 1$

29. Let $a \geq 2$. The integer $a^m + 1$ is a prime. Then :

(A) a is even and m is a power of 2

(B) a is a power of 2 and m is odd

(C) a is odd and m is even

(D) Cannot say anything

30. If $2^{65} \equiv b \pmod{19}$, then b is :

(A) 4

(B) 1

(C) 9

(D) 6

31. Last two decimal digits of 3^{1492} are :

(A) 14

(B) 41

(C) 10

(D) 40

32. Which of the following equation have only finitely many solutions (integral) ?

(A) For any $a \in \mathbb{Z}$, $ax + (a + 1)y = 3$

(B) For any $a \in \mathbb{Z}$, $ax + a^2y = a^{11}$

(C) For any $a \in \mathbb{Z}$, $9x + 3y = a^3 - a$

(D) For any $a \in \mathbb{Z}$, $(a + 1)x + (a - 1)y = 3$

33. While writing numbers from 1 to 10000 how many times the digit 9 will be written ?

(A) 4000

(B) 3600

(C) 4100

(D) 3000

34. The set of all trancendental numbers is an :

(A) Countable set

(B) Uncountable set

(C) Finite set

(D) Null set

35. Let X be the set of all real valued continuous functions defined on the closed interval [a, b]. Define the mapping d_∞ and d_1 on X × X into $\mathbb{R}$ as follows :

$$d_\infty (x, y) = \max_{t \in [a, b]} |x(t) - y(t)|$$

and $d_1(x, y) = \int_a^b |x(t) - y(t)| dt.$

Then :

(A) d_∞ and d_1 are metrics on X

(B) d_∞ and d_1 are not metrics on X

(C) d_∞ is a metric and but not d_1

(D) d_1 is a metric and but not d_∞

36. ℓ^∞ ,the space of all bounded sequences in $\mathbb{R}$ is not :

(A) Complete

(B) Separable

(C) Divergent

(D) None of these

37. A metric space X is said to be separable if :

(A) It has a countable subset which is dense in X

(B) It has a uncountable subset which is dense in X

(C) It has a countable subset which is not dense in X

(D) It has a uncountable subset which is not dense in X

38. In $\mathbb{R}$ with the usual metric, which of the following statement is true ?

(A) The set of integers dense in $\mathbb{R}$

(B) The set of rationals dense in $\mathbb{R}$

(C) Cantor set is nowhere dense in $\mathbb{R}$

(D) None of these

39. Every complete metric space is of :

(A) First category

(B) Second category

(C) Both (A) and (B)

(D) None of these