DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO

TEST BOOKLET

Sl. No. 01811

Subject Code : 18

Subject : Mathematics

LECTURERS FOR NON-GOVT. AIDED COLLEGES OF ODISHA

Time Allowed : 3 Hours

Maximum Marks : 165

: INSTRUCTIONS TO CANDIDATES :

1. **IMMEDIATELY AFTER THE COMMENCEMENT OF THE EXAMINATION, YOU SHOULD CHECK THAT THIS TEST BOOKLET CONTAINS 31 PAGES AND DOES NOT HAVE ANY UNPRINTED OR TORN OR MISSING PAGES OR ITEMS ETC. IF SO, GET IT REPLACED BY A COMPLETE TEST BOOKLET.**

2. You have to enter your **Roll No.** on the Test Booklet in the Box provided alongside. **DO NOT** write anything else on the Test Booklet.

3. The Test Booklet contains **165** questions. Each question comprises four answers. You have to select the correct answer which you want to mark (darken) on the Answer Sheet. In case, you feel that there is more than one correct answer, you should mark (darken) the answer which you consider the best. In any case choose ONLY ONE answer for each question. If more than one answer is darkened it will be considered as wrong.

4. You have to mark (darken) all your answers **ONLY** on the **separate OMR Answer Sheet** provided, by **using BLACK BALL POINT PEN**. You have to do rough work on the space provided in the Test Booklet only. See instruction in the Answer Sheet.

5. All questions carry equal marks, i.e. of one mark for each correct answer and each wrong answer will result in negative marking of **0.25** mark.

6. Before you proceed to mark (darken) in the Answer Sheet the answers to various questions in the Test Booklet, you have to fill in some particulars in the Answer Sheet as per the instructions in your Admit Card.

7. After you have completed filling in all your answers on the Answer Sheet and after completion of the examination, you should hand over to the Invigilator the **Original Answer Sheet (OMR Answer Sheet)** issued to you. You are allowed to take with you the candidate's copy/second page of the Answer Sheet along with the Test Booklet after completion of the examination for your reference.

Candidate's full signature

Invigilator's signature

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2018

1. Which of the following is not correct ?

(A) The set $\mathbb{Q}^+$ of positive rationals is a group under ordinary multiplication.

(B) The subset {1, – 1, i, –i} of complex numbers is a group under complex multiplication.

(C) The set $\mathbb{Z}_n = \{0, 1, 2, ...n-1\}$ for $n \geq 1$ is a group under addition modulo n.

(D) The set {0, 1, 2, 3} is a group under multiplication modulo 4.

2. The set $\mathbb{Z}_n = \{0, 1, 2, ..., n-1\}$ is a group under multiplication modulo n if and only if :

(A) n is a prime

(B) n is even

(C) n is odd

(D) n is not a prime

3. Let G be a group and let a be an element of order n in G. If $a^k = e$, then :

(A) n divides k

(B) k divides n

(C) n does not divide k

(D) n and k are primes

4. The order of (123)(145) in the permutation group S_5 is :

(A) 6

(B) 3

(C) 5

(D) 9

5. The group of even permutations of n symbols is denoted by A_n and it is called alternating group of degree n. For n > 1, A_n has order :

(A) n!

(B) $\frac{(n+1)!}{2}$

(C) $\frac{(n-1)!}{2}$

(D) $\frac{n!}{2}$

6. For every integer a and every prime p :

(A) $p^a \bmod p = p \bmod a$

(B) $a^p \bmod p = a \bmod p$

(C) $a^p \bmod p = p \bmod p$

(D) $a^p \bmod p = p \bmod a$

7. The group of rotations of a cube is isomorphic to :

(A) A_4

(B) S_5

(C) S_4

(D) A_5

8. An integral domain is a commutative ring with unity and :

(A) Zero-divisors

(B) No zero-divisors

(C) Zero multipliers

(D) None of these

9. The characteristic of an integral domain is :

(A) 0 or non-prime

(B) 0 or even number

(C) 0 or odd number

(D) 0 or prime

10. If F is a field of characteristics 0, then F contains a subfield isomorphic to the :

(A) Irrational numbers

(B) Rational numbers

(C) Even numbers

(D) Odd numbers

11. The polynomial $3x^5 + 15x^4 - 20x^3 + 10x + 20$ is irreducible over :

(A) $\mathbb{R}$

(B) $\mathbb{R} - \mathbb{Q}$

(C) $\mathbb{Q}$

(D) $\mathbb{Z}$

12. Let G be a group and a, b $\in$ G such that o(a) = 6, o(b) = 2 and $a^3b = ba$. Then o(ab) is :

(A) 6

(B) 8

(C) 12

(D) 2

13. The number of subgroups of order 2 in the permutation group S_3 is :

(A) 1

(B) 3

(C) 12

(D) 2

14. If G is an abelian group, then the number of conjugacy classes equal to :

(A) o(G)

(B) 1

(C) $o(G) - o(\mathbb{Z}(G))$

(D) 2

15. Let G be an abelian group with the identity e. Which one of the following statement is true ?

(A) $H = \{x \in G : \text{order of } x \text{ is odd}\}$ is a subgroup of G

(B) $H = \{x \in G : \text{order of } x \text{ is even}\} \cup \{e\}$ is a subgroup of G

(C) Every subgroup of G is normal

(D) G is cyclic

16. In the ring $\mathbb{Z}_8[x]$, the element $4x^2 + 6x + 3$ is :

(A) A nilpotent

(B) A unit

(C) A idempotent

(D) A non-zero divisor

17. Suppose that $\phi : \mathbb{Z}_{20} \rightarrow \mathbb{Z}_{20}$ is an automorphisim such that $\phi(5) = 5$, the number of possibilities for $\phi(1)$ is :

(A) 4

(B) 1

(C) 5

(D) 20

18. Let S_3 be the permutation group of {1, 2, 3}. Then there exists a non-trivial group homomorphism $f : S_3 \rightarrow S_3$ such that :

(A) Kernel f = {(12), e}

(B) Kernel f = {(123), (132), e}

(C) Kernel f = {(123), (12)}

(D) None of these

19. Let A be a 5 × 5 real matrix. Suppose 0 is one of eigenvalues of A. Which of the following statement is true ?

(A) System Ax = 0 has unique solution

(B) System Ax = C has unique solution for any C

(C) System Ax = 0 has a non-trivial solution

(D) None of these

20. Which of the following subset is a subspace of the vector space $\mathbb{R}^3$ over the field $\mathbb{R}$?

(A) $\{(u, v, w) \in \mathbb{R}^3 : 2u + 3v + 4w = 0\}$

(B) $\{(u, v, w) \in \mathbb{R}^3 : 2u + 3v + 4w = 1\}$

(C) $\{(u, v, w) \in \mathbb{R}^3 : u > 0, v > 0, w > 0)$

(D) $\{(u, v, w) \in \mathbb{R}^3$: u, v, w are rationals}

21. The dimension of the vector space $\mathbb{Q}[\sqrt{2}]$ over the field $\mathbb{Q}$ is:

(A) 1

(B) ∞

(C) 4

(D) 2

22. Let X and Y be subspaces of finite dimensional vector space V. Let $X + Y = \{x + y : x \in X, y \in Y\}$. The dimension of the subspace X + Y is always equal to :

(A) $\dim (X + Y) = \dim(X) + \dim (Y) - \dim (X \cap Y)$

(B) $\dim (X + Y) = \dim(X) + \dim (Y)$

(C) $\dim (X + Y) = \max\{\dim(X), \dim (Y)\}$

(D) $\dim (X + Y) = \dim(X) + \dim (Y) + \dim (X \cap Y)$

23. Suppose G is a finite group and H is a subgroup of G. If $[G : H] = 2$, then which of the following statement is true ?

(A) If $x \in H$ and $y \notin H$, then $xy \in H$

(B) If $x \notin H$ and $y \notin H$, then $xy^{-1} \in H$

(C) If $x \notin H$ and $y \notin H$, then $xy \in H$

(D) Both (B) and (C) are true

24. The characteristic polynomial of the 3×3 matrix $A = \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}$ is given by :

(A) $-\lambda^3 + (a + b + c)\lambda^2 - (ab + bc + ca)\lambda + abc$

(B) $\lambda^3 + (a + b + c)\lambda^2 - (ab + bc + ca)\lambda + abc$

(C) $-\lambda^3 - (a + b + c)\lambda^2 - (ab + bc + ca)\lambda + abc$

(D) $\lambda^3 + (a + b + c)\lambda^2 + (ab + bc + ca)\lambda + abc$

25. The greatest common divisor (gcd) of $5n + 3$ and $7n + 4$, for all $n \in \mathbb{N}$ is :

(A) 1

(B) 5

(C) n

(D) 2

26. How many three digit numbers are divisible by 6 ?

(A) 142

(B) 150

(C) 148

(D) 166

27. If gcd (1492, 1066) = 2, then we have ℓcm (1492, 1066) = ?

(A) 752936

(B) 795326

(C) 795236

(D) Can not say

28. Let $a, b \in \mathbb{Z}$ such that gcd (a, b) = 5. Then the equation $ax + by = c^2$ has :

(A) Unique solution for any $c \in \mathbb{Z}$

(B) Infinitely many solutions if $(25, c^2) = 1$

(C) No solution for any $c \in \mathbb{Z}$

(D) Infinitely many solutions $(x,y) \in \mathbb{Z} \times \mathbb{Z}$ if $(25, c^2) \neq 1$

29. Let $a \geq 2$. The integer $a^m + 1$ is a prime. Then :

(A) a is even and m is a power of 2

(B) a is a power of 2 and m is odd

(C) a is odd and m is even

(D) Cannot say anything

30. If $2^{65} \equiv b \pmod{19}$, then b is :

(A) 4

(B) 1

(C) 9

(D) 6

31. Last two decimal digits of 3^{1492} are :

(A) 14

(B) 41

(C) 10

(D) 40

32. Which of the following equation have only finitely many solutions (integral) ?

(A) For any $a \in \mathbb{Z}$, $ax + (a + 1)y = 3$

(B) For any $a \in \mathbb{Z}$, $ax + a^2y = a^{11}$

(C) For any $a \in \mathbb{Z}$, $9x + 3y = a^3 - a$

(D) For any $a \in \mathbb{Z}$, $(a + 1)x + (a - 1)y = 3$

33. While writing numbers from 1 to 10000 how many times the digit 9 will be written ?

(A) 4000

(B) 3600

(C) 4100

(D) 3000

34. The set of all trancendental numbers is an :

(A) Countable set

(B) Uncountable set

(C) Finite set

(D) Null set

35. Let X be the set of all real valued continuous functions defined on the closed interval [a, b]. Define the mapping d_∞ and d_1 on X × X into $\mathbb{R}$ as follows :

$$d_\infty(x, y) = \max_{t \in [a, b]} |x(t) - y(t)|$$

and $d_1(x, y) = \int_a^b |x(t) - y(t)|dt.$

Then :

(A) d_∞ and d_1 are metrics on X

(B) d_∞ and d_1 are not metrics on X

(C) d_∞ is a metric and but not d_1

(D) d_1 is a metric and but not d_∞

36. ℓ^∞ ,the space of all bounded sequences in $\mathbb{R}$ is not :

(A) Complete

(B) Separable

(C) Divergent

(D) None of these

37. A metric space X is said to be separable if :

(A) It has a countable subset which is dense in X

(B) It has a uncountable subset which is dense in X

(C) It has a countable subset which is not dense in X

(D) It has a uncountable subset which is not dense in X

38. In $\mathbb{R}$ with the usual metric, which of the following statement is true ?

(A) The set of integers dense in $\mathbb{R}$

(B) The set of rationals dense in $\mathbb{R}$

(C) Cantor set is nowhere dense in $\mathbb{R}$

(D) None of these

39. Every complete metric space is of :

(A) First category

(B) Second category

(C) Both (A) and (B)

(D) None of these

40. Let $X \subset \mathbb{N}$ be a non-empty finite set and $Y \subset \mathbb{N}$ be an infinite set. Define $A = \{x - y : x \in X \text{ and } y \in Y\}$. Then :

(A) $\text{Inf}(A) = -\infty$ and $\sup(A) = \infty$

(B) $\text{Inf}(A) = -\infty$ and $\sup(A) < \infty$

(C) $\text{Inf}(A) > -\infty$ and $\sup(A) = \infty$

(D) $\text{Inf}(A) > -\infty$ and $\sup(A) < \infty$

41. Let X be a non-empty set and A, $B \subset X$. If $(A \cup B) - (A \cap B)$ is a finite set, then :

(A) Both A and B are finite set

(B) Atleast one of the A, B is a finite set

(C) X is a finite set

(D) None of these

42. Let $f, g : \mathbb{R} \to \mathbb{R}$ be are polynomials with deg(f) = 5 and deg (g) = 7. Let $A = \{x \in \mathbb{R} : f(x) = g(x)\}$. Then :

(A) $|A| = 5$

(B) $|A| \leq 7$

(C) A is finite, but $|A|$ cannot be arbitrarily large

(D) A can be finite

43. If $0 < a < 1$, then $\lim_{n\to\infty} (a^n)$ is :

(A) 0

(B) 1

(C) ∞

(D) a^{∞}

44. If $X = (x_n)$ be a convergent sequence of real numbers, then X is a :

(A) Not a Cauchy sequence

(B) A Cauchy sequence

(C) Unbonded

(D) None of these

45. Which of the following statement is not true ?

(A) A Cauch sequence of real numbers is bounded

(B) A sequence of real number is convergent if and only if it is a Cauchy sequence

(C) Every contractive sequence is a Cauchy sequence

(D) The sequence $1 + (-1)^n$ is a Cauchy sequence

46. The p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is :

(A) Converges when $p > 1$ and diverges when $0 < p \leq 1$

(B) Diverges when $p > 1$ and converges when $0 < p \leq 1$

(C) Converges for all p

(D) Diverges for all p

47. The series is $\sum_{n=1}^{\infty} \frac{1}{n!}$ is :

(A) Convergent

(B) Divergent to ∞

(C) Oscillatory

(D) Divergent to $-\infty$

48. Let $A \subset \mathbb{R}$. Let $f, g : A \to \mathbb{R}$ and let $c \in \mathbb{R}$ a cluster point of A. Suppose that $f(x) \leq g(x)$ for all $x \in A$, $x \neq a$, implies that :

(A) If $\lim_{x \to a} f = \infty$, then $\lim_{x \to a} g$ is finite

(B) If $\lim_{x \to a} f = \infty$, then $\lim_{x \to a} g = \infty$

(C) If $\lim_{x \to a} f = -\infty$, then $\lim_{x \to a} g = +\infty$

(D) If $\lim_{x \to a} g = -\infty$, then $\lim_{x \to a} g$ is finite

49. Suppose $f : \mathbb{R} \to \mathbb{R}$ is continuous on $\mathbb{R}$ and $f(x) = 0$ for every rational number x, then :

(A) $f(x) = 0$ for all $x \in \mathbb{R}$

(B) $f(x) = 0$ only for all $x \in \mathbb{Z}$

(C) $f(x) = 0$ only for all $x \in \mathbb{N}$

(D) None of these

50. Let $I = [a, b]$ and $f : I \to \mathbb{R}$ be continuous on I. If $f(a) < 0 < f(b)$, or if $f(a) > 0 > f(b)$, then there exists a number $c \in (a, b)$ such that :

(A) $f'(c) = 0$

(B) $f(c) = k, k > 0$

(C) $f(c) = 0$

(D) $f'(c) = k, k > 0$

51. Which of the following statement is not correct ?

(A) If $f : A \to \mathbb{R}$ is a Lipschitz function, then f is uniformly continuous on A

(B) If $f : A \to \mathbb{R}$ is uniformly continuous on a subset A of $\mathbb{R}$ and if (x_n) is a Cauchy sequence in A., then $f(x_n)$ is not a Cauchy sequence in $\mathbb{R}$

(C) If f and g are each uniformly continuous on $\mathbb{R}$, the composite function fog is uniformly continuous on $\mathbb{R}$

(D) If f is uniformly continuous on a bounded subset A of $\mathbb{R}$,then f is bounded on A

52. Let $f : \mathbb{R} \to \mathbb{R}$ defined by the series

$f(x) = \sum_{n=0}^{\infty} \frac{1}{2^n} \cos(3^n x)$. Then f :

(A) Is continuous at every point but derivative exists at finite number of points

(B) Is not continuous at every points

(C) Is differentiable at every points

(D) Is continuous at every point but whose derivative does not exists anywhere

53. If $f : [a, b] \to \mathbb{R}$ is a differentiable function and if k is a number between $f'(a)$ and $f'(b)$, then :

(A) There is atmost one point $c \in (a, b)$ such that $f'(c) = k$

(B) There is at least one point $c \in (a, b)$ such that $f'(c) = k$

(C) There is no point $c \in (a, b)$ such that $f'(c) = k$

(D) None of these

54. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x + 2x^2 \sin\frac{1}{x}$ for $x \neq 0$ and $f(0) = 0$. Then $f'(0)$ is :

(A) 0

(B) –1

(C) 1

(D) ∞

55. The value of

$\lim_{n \to \infty} \frac{1}{n}\left(1 + \frac{1}{2} + \cdots\cdots + \frac{1}{n}\right)$ is

(A) 0

(B) 1

(C) ∞

(D) e

56. The Lebesgue measure of $\mathbb{R}$ is :

(A) Finite

(B) 0

(C) Infinite

(D) None of these

57. Which of the following statement is true ?

(A) Lebesgue measure is not translation invariant

(B) Open intervals are Lebesgue measurable

(C) Lebesgue measure of Cantor set non-zero

(D) The Cantor set has countably many elements

58. Let μ be a measure on a set Ω and let $f, g : \Omega \rightarrow [0, \infty]$ measurable. Then which of the following statement is false ?

(A) If $A \subset \Omega$ is measurable and $f(x) = 0$ for almost every $x \in A$, then $\int_A f d\mu = 0$

(B) If $A \subset \Omega$ is a null set, then $\int_A f d\mu = 0$

(C) If $f \leq g$, then $\int_A f d\mu \leq \int_A g d\mu$

(D) If $A \subset B \subset \Omega$ are measurable, then $\int_A f d\mu \geq \int_A g d\mu$

59. A measure μ on a metric space (Ω, d) is called a Borel measure, if :

(A) $\mu(\Omega) < \infty$

(B) All Borel sets are measurable

(C) If for every $x \in \Omega$, there is a $r > 0$ such that $\mu(B_r(x)) < \infty$

(D) None of these

60. Let f_1, f_2 ...... be a measurable functions such that $f_n \rightarrow f$ μ-a.e.there exists a μ-summable function g such that $|f_n| \leq g$, then :

$$\lim_{n\to\infty} \int |f_n - f| d\mu = 0 \text{ and } \lim_{n\to\infty} \int f_n d\mu = \int f d\mu$$

This represent as :

(A) Fatou's Lemma

(B) Monotone converge theorem

(C) Dominated convergence theoem

(D) None of there

61. The function $f(x) = -\log(|x|)$ is integrable over any compact interval in the sense of :

(A) Both Riemann and Lebesgue

(B) Riemann only

(C) Lebesgue only

(D) None of these

62. Every countable set has measure :

(A) 1

(B) 0

(C) Non-zero

(D) Uncountable

63. Which of the following statement is wrong ?

(A) If f is a measurable function and let fog = 0 a.e., then g is measurable

(B) Monotone functions are not measurable

(C) If f is a measurable function and ess sup p | f | < ∞ , then f is essentially bounded

(D) If f is a measurable function, then ess sup $f \leq$ sup f.

64. The value of $\int_1^\infty \frac{1}{x}\,dx$ is :

(A) 0

(B) 1

(C) ∞

(D) None of these

65. Let $f \in BV[a, b]$ (f is a function of bounded variation on [a, b]) and $x \in (a, b)$, then :

(A) $f(x^-)$ exists but $f(x^+)$ does not exists

(B) $f(x^+)$ and $f(x^-)$ exist

(C) $f(x^-)$ does not exists but $f(x^+)$ exists

(D) We can not say anything about $f(x^+)$ and $f(x^-)$

66. BV [a, b] is a vector space over :

(A) Real numbers

(B) Complex numbers

(C) Rationals only

(D) Irrationals only

67. The function $f(x) = |x|^p$, $0 < p < 1$ is :

(A) Not Lipschitz at $x = 0$

(B) Lipschitz at $x = 0$ with Lipschitz constant 1

(C) Lipschitz at $x = 0$

(D) Differentiable at $x = 0$

68. The solution of $y' + y = xy^4$ is given by :

(A) $y^{-3} = x + \frac{1}{3} + ce^{3x}$

(B) $y^3 = x + \frac{1}{3} + ce^{3x}$

(C) $y^{-3} = x + \frac{1}{3} + ce^{-3x}$

(D) $y^3 = x + \frac{1}{3} + ce^{-3x}$

69. The general solution of $(x^2y + 1)dx + (\frac{1}{2}y + \frac{1}{3}x^3)\,dy = 0$ passing through $(\alpha, 0)$ is given by :

(A) $x - \frac{1}{4}y^2 + \frac{1}{3}x^3y = \alpha$

(B) $x + \frac{1}{4}y^2 + \frac{1}{3}x^3y = \alpha$

(C) $x + \frac{1}{4}y^2 - \frac{1}{3}x^3y = \alpha$

(D) $-x + \frac{1}{4}y^2 + \frac{1}{3}x^3y = \alpha$

70. The initial value problem $\frac{dx}{dt} = x^{3/2}(t)$, $x(0) = 0$ has :

(A) Unique solution

(B) Two solution

(C) Infinitely many solutions

(D) None of these

71. If x_1 and x_2 are any solutions of $x'' + p(t)x' + q(t)x = 0$ on a given interval I, where p(t) and q(t) are continuous, then Wronskian of x_1 and x_2 is given by :

(A) $W(t) = ce^{\int p(t)dt}$

(B) $W(t) = -ce^{\int p(t)dt}$

(C) $W(t) = ce^{-\int p(t)dt}$

(D) 0

72. The general solution of $x'' - 2x' + x = e^t$ is given by :

(A) $x = (c_1 + c_2 t)e^t + \frac{1}{4}e^{-t}$

(B) $x = (c_1 + c_2 t)e^{-t} + \frac{1}{4}e^{-t}$

(C) $x = (c_1 + c_2 t)e^{-t} - \frac{1}{4}e^{-t}$

(D) $x = (c_1 + c_2 t)e^t + \frac{1}{4}e^t$

73. The value of $\frac{1}{D^2 + a^2}$ cos at, is given by :

(A) $\frac{t}{2a}$ sin at

(B) $\frac{t}{2a}$ cos at

(C) $-\frac{t}{2a}$ sin at

(D) $-\frac{t}{2a}$ cos at

74. If P_n is the Legendre polynomial of degree n, then $\int_{-1}^{1} [P_n(x)]^2 dx$ is given by :

(A) $\frac{2}{n+1}$, n = 0, 1, 2, ...

(B) $\frac{1}{2n+1}$, n = 0, 1, 2, ...

(C) $\frac{2n}{2n+1}$, n = 0, 1, 2, ...

(D) $\frac{2n}{2n+1}$, n = 0, 1, 2, ...

75. The Bessel's function $J_p(x)$ for p = 1/2 is given by :

(A) $\sqrt{\frac{2}{\pi}}$ sin x

(B) $\sqrt{\frac{2}{\pi x}}$ sin x

(C) $\sqrt{\frac{2}{\pi x}}$ cos x

(D) $\sqrt{\frac{2}{\pi}}$ cos x

76. The inverse Laplace transform of

$\dfrac{p+7}{p^2+2p+5}$ is given by :

(A) $e^{t}(\cos 2t + 3\sin 2t)$

(B) $e^{t}(\sin 2t + 3\cos 2t)$

(C) $e^{-t}(\cos 2t + 3\sin 2t)$

(D) $e^{-t}(\sin 2t + 3\cos 2t)$

77. The Laplace transform of a periodic function f(t) with period T > 0 is :

(A) $\dfrac{1}{1-e^{-sT}}\int_0^T e^{-st}f(t)\,dt$

(B) $\dfrac{1}{1-e^{-sT}}\int_0^T e^{st}f(t)\,dt$

(C) $\dfrac{1}{1-e^{sT}}\int_0^T e^{-st}f(t)\,dt$

(D) $\dfrac{1}{1-e^{sT}}\int_0^T e^{st}f(t)\,dt$

78. The convolution of $f(t) = t^2$ and $g(t) = \sin t$ is given by :

(A) $f * g = \dfrac{t^3}{2}\sin t$

(B) $f * g = t^3 \sin t$

(C) $f * g = \dfrac{t^3}{3}\sin t$

(D) $f * g = \dfrac{t^3}{3}\cos t$

79. The eigen functions of the Sturm-Liouville problem $\dfrac{d^2y}{dx^2} + \lambda y = 0$, $y(0) = 0$, $y'(\pi) = 0$ is given by :

(A) $y_n = c_n \sin\dfrac{(2n-1)}{2}x$, n = 1, 2, 3, ...

(B) $y_n = c_n \cos\dfrac{(2n-1)}{2}x$, n = 1, 2, 3, ...

(C) $y_n = c_n \sin\dfrac{(2n+1)}{2}x$, n = 1, 2, 3, ...

(D) $y_n = c_n \cos\dfrac{(2n+1)}{2}x$, n = 1, 2, 3, ...

80. All the eigen-values of regular Sturm-Liouville Problem are :

(A) 0

(B) Complex

(C) Real

(D) None of these

81. If u is a function of x, y and z satisfies the partial differential equation

$$(y-z)\frac{\partial u}{\partial x}+(z-x)\frac{\partial u}{\partial y}+(x-y)\frac{\partial u}{\partial z}=0$$

is given by :

(A) $u = f(xy + yz + zx, x^2 + y^2 + z^2)$

(B) $u = f(xyz, x^2 + y^2 + z^2)$

(C) $u = f(x + y + z, x^2y^2z^2)$

(D) $u = f(x + y + z, x^2 + y^2 + z^2)$

82. The complete integral of the equation $pq = 1$, where $\frac{\partial z}{\partial x} = p$, $\frac{\partial z}{\partial y} = q$ is given by :

(A) $z = ax + \frac{y}{a} + b$

(B) $z = ax - \frac{y}{a} + b$

(C) $z = ax - \frac{y}{a} - b$

(D) $z = -ax + \frac{y}{a} + b$

83. The complete integral of the equation

$$(p + q)(z - xp - yq) = 1$$

where $\frac{\partial z}{\partial x} = p$, $\frac{\partial z}{\partial y} = q$ is given by :

(A) $z = ax + by + \frac{1}{a-b}$

(B) $z = ax + by + \frac{1}{a+b}$

(C) $z = ax - by + \frac{1}{a+b}$

(D) None of these

84. The orthogonal trajectories of the family of rectangular hyperbolas $y = \frac{c_1}{x}$ is :

(A) $y^2 - x^2 = c$

(B) $y^2 + x^2 = c$

(C) $x^2y^2 = c$

(D) $\frac{x^2}{y^2}$

85. A particular integral of the equation $(D^2 - D')z = 2y - x^2$ is given by :

(A) xy^2

(B) x^2y

(C) x^2y^2

(D) 0

86. The one dimensional wave equation

$$\frac{\partial^2 z}{\partial x^2} = c^2 \frac{\partial^2 z}{\partial y^2},$$ (where c is a constant) is :

(A) Parabolic

(B) Hyperbolic

(C) Elliptic

(D) None of these

87. The solution of $\frac{\partial^2 z}{\partial x \partial y} = 4xe^{2y}$ is given by :

(A) $z = xe^{2y} + \xi_1(x) + \xi_2(y)$

(B) $z = x^2e^y + \xi_1(x) + \xi_2(y)$

(C) $z = x^2e^{2y} + \xi_1(x) + \xi_2(y)$

(D) $z = xe^y + \xi_1(x) + \xi_2(y)$

88. The Laplace transform of $t^{1/2}$ is :

(A) $\frac{\sqrt{\pi}}{2s^{3/2}}$

(B) $\frac{\sqrt{\pi}}{s^{3/2}}$

(C) $\frac{\sqrt{\pi}}{2s^{1/2}}$

(D) $\frac{\sqrt{\pi}}{s^{1/2}}$

89. The solution of the Initial Value Problem (IVP) ;

$u_{tt} = c^2 u_{xx}, -\infty < x, t < \infty$

with $u(x, 0) = \sin x$ and $u_t(x, 0) = 0$ is given by :

(A) $u(x, t) = \frac{1}{2}[\cos(x - ct) + \cos(x + ct)]$

(B) $u(x, t) = \frac{1}{2}[\sin(x - ct) + \cos(x + ct)]$

(C) $u(x, t) = \frac{1}{2}[\sin(x - ct) + \sin(x + ct)]$

(D) $u(x, t) = \frac{1}{2}[\cos(x - ct) + \sin(x + ct)\}$

90. Let Ω be a bounded domain in $\mathbb{R}^2$ with boundary $\partial\Omega$. The solution of the Dirchlet's Problem $\Delta u(x, y) = -f(x, y)$ in Ω, with $u(x, y) = g(x, y)$ on $\partial\Omega$. If it exists then it is :

(A) Not unique

(B) Unique

(C) Infinitely many solution

(D) Trivial solution

91. The solution of the Cauchy Problem $(1 + x^2)u_x + xyu_y = 0,\ u(0, y) = y^2$ is given by :

(A) $\frac{x^2}{1+y^2}$

(B) $\frac{x}{1+y^2}$

(C) $\frac{y^2}{1+x^2}$

(D) $\frac{y}{1+x^2}$

92. The rate of convergence of Secant method for finding the roots of $f(x) = 0$ is :

(A) 1.618

(B) 1.168

(C) 1.816

(D) 2

93. Given $f(2) = 4$, $f(2.5) = 5.5$, the linear interpolating polynomial by using Lagrange's interpolation is :

(A) $3x + 2$

(B) $3x - 2$

(C) $3x - 1$

(D) $3x + 1$

94. If $g(x)$ is a continuous function on some interval $[a, b]$ and differentiable on (a, b) and if $g(a) = 0$ and $g(b) = 0$, then there is at least one point c inside (a, b) for which :

(A) $g'(c) \neq 0$

(B) $g(c) = 0$

(C) $g'(c) = 0$

(D) $g(c) \neq 0$

95. The unique polynomial of degree 2 or less, such that $f(0) = 1$, $f(1) = 3$, $f(3) = 55$, using Newton's divided difference interpolation is :

(A) $8x^2 - 6x + 1$

(B) $8x^2 + 6x + 1$

(C) $8x^2 - 6x - 1$

(D) $8x^3 - 6x - 1$

96. The relation between Central difference operator δ and Shift operator is :

(A) $\delta = E^{1/2} - E^{-1/2}$

(B) $\delta = \frac{1}{2}(E^{1/2} + E^{-1/2})$

(C) $\delta = 1 - E^{-1}$

(D) $\delta = 1 + E^{-1}$

97. The Chebyshev polynomials of the first kind $J_n(x)$ defined on [1, −1] is orthogonal with respect weight function :

(A) $\frac{1}{\sqrt{1+x^2}}$

(B) $\frac{1}{\sqrt{1-x^2}}$

(C) $\frac{1}{\sqrt{1-x}}$

(D) $\frac{1}{\sqrt{1+x}}$

98. The approximate value of $\int_0^1 \frac{dx}{1+x}$ by using Gauss-Legendre three point formula is given by :

(A) 0.693122

(B) 0.693100

(C) 0.639122

(D) 0.693

99. The Hermite Polynomials are orthogonal with respect to the weight funtion e^{-x^2} on :

(A) $(0, \infty)$

(B) $(-\infty, 0)$

(C) $(-\infty, \infty)$

(D) None of these

100. Which of the following iterative method is more efficient for solving simultaneous equations ?

(A) Jacobi's method

(B) Gauss-Seidal method

(C) Relaxation method

(D) None of these

101. A lattice is a partially ordered set in which :

(A) $a \wedge b = \inf(a, b)$ and $a \vee b = \sup(a, b)$ exist for any pair of elements a and b

(B) $a \wedge b = \inf(a, b)$ exist only

(C) $a \vee b = \sup(a, b)$ exist only

(D) None of these

102. Every finite Lattice L is :

(A) Unbounded

(B) Bounded

(C) Bounded above only

(D) Bounded below only

103. Let $\mathbb{N} = \{1, 2, 3, ...\}$ be ordered by divisibility. Which of the following statement is ture ?

(A) $\{24, 2, 6\}$ is linearly ordered

(B) $\{3, 15, 5\}$ is linearly ordered

(C) $(2, 8, 32, 4\}$ is not linearly ordered

(D) None of these

104. A Boolean algebra B is a :

(A) Bounded, distributive and complemented lattice

(B) Unbounded, distributive and complemented lattice

(C) Bounded, commutative and complemented lattice

(D) Bounded, non-commulative and complemented lattice

105. For some positive integer n, finite Boolean algebra has :

(A) 2^n elements

(B) n! elements

(C) $2^{n!}$ elements

(D) 2^{n+1} elements

106. Let T be a finite non-empty tree, then :

(A) T is non-planar

(B) T is a planar

(C) T has one less vertex than edge

(D) None of these

107. Let a finite non-empty connected plannar graph have vertex, edge and face sets V, E and F respectively, then :

(A) $|V| + |F| = |E| - 1$

(B) $|V| - |F| = |E| - 1$

(C) $|V| + |F| = |E| + 1$

(D) None of these

108. The number of faces in any two plane representations of the same finite planar graphs are :

(A) Not equal

(B) Equal

(C) 0

(D) None of these

109. Any finite planar graph G can be coloured using :

(A) Six colours

(B) Five colours or fewer

(C) Seven colours

(D) None of these

110. In any graph :

(A) There is an odd number of vertices of even degree

(B) There is an even number of vertices of even degree

(C) There is an odd number of vertices of odd degree

(D) There is an even number of vertices of odd degree

111. There is an Eulerian circuit in an finite connected graph if and only if all its vertices have :

(A) Odd degree

(B) Both odd and even degree

(C) Even degree

(D) None of these

112. If a partially ordered set $(P, \leq)$ has atleast $n^2 + 1$ elements, then it has a totally ordered subset of size :

(A) $n - 1$

(B) n

(C) $n + 1$

(D) $n!$

113. Which of the following graphs are not connected ?

(A) $V = \mathbb{Z}$, m, n joined if $m - n$ is even

(B) $V = \mathbb{Z}$, m, n joined if $m - n$ is prime

(C) $V = \mathbb{Z}$, m, n joined if $|m - n| = 5$ or 7

(D) None of these

114. In a finite loop-free graph, the sum of the degrees of the vertices is equal to :

(A) 0

(B) Infinite

(C) Thrice the number of edges

(D) Twice the number of edges

115. A graph is bipartite if and only if it has no circuit of :

(A) Even length

(B) Odd length

(C) Both even and odd length

(D) None of these

116. A feasible solution to an L.P. problem :

(A) Must satisfy all of the problem's constraints simultaneously

(B) Need not satisfy all of the constraints, only some of them

(C) Must be corner point of the feasible region

(D) Must optimize the value of the objective function

117. Which of the following is a valid objective function for linear programming problem ?

(A) Max 5xy

(B) Min $4x + 3y + \frac{2}{3}$

(C) Max $5x^2 + 6y^2$

(D) Min $(x_1 + x_2)/x_3$

118. The North West corner rule :

(A) Is used to find initial feasible solution

(B) Is used to find an optimal solution

(C) Is based on the concept of minimizing opportunity cost

(D) None of these

119. In simplex method, slack, surplus and artificial variables are restricted to be :

(A) Multiplies

(B) Negative

(C) Non-negative

(D) Divided

120. According to algebra of simplex method, slack variables assigned zero coefficients because :

(A) No contribution in objective function

(B) High contribution in objective function

(C) Divisor contribution in objective function

(D) Less contribution in objective function

121. Which of the following is a method for improving an initial solution in a transportation problem ?

(A) Stepping-Stone

(B) North West Corner

(C) South East Corner

(D) Intuitive Lowest Cost

122. Variable in dual problem which can assume negative values, positive values or zero values is classified as :

(A) Unrestricted constant

(B) Restricted constant

(C) Restricted variable

(D) Unrestricted variable

123. The assignment model is a special case of the :

(A) Maximum flow model

(B) Transportation model

(C) Shortest-route model

(D) None of these

124. Who coined the term Operation Research ?

(A) J. F. Mc Closkey

(B) F. N. Trefethen

(C) F. M. Adams

(D) Both (A) and (B)

125. Which of the following models consider as one of the important variable ?

(A) Static models

(B) Dynamic models

(C) Both (A) and (B)

(D) None of these

126. MODI method is used to obtain :

(A) Optimal solution

(B) Optimal test

(C) Both (A) and (B)

(D) Optimization

127. A basic solution which also satisfies the condition in which all the basic variables are non-negative is called as :

(A) Basic feasible solution

(B) Feasible solution

(C) Optimal solution

(D) None of these

128. The method used to solve an assignment problem is :

(A) Hungarian

(B) American

(C) German

(D) All are incorrect

129. Optimal solution is a feasible solution (not necessarily basic) which minimizes the :

(A) Time taken

(B) Partial cost

(C) Total cost

(D) None of these

130. VAM stands for :

(A) Vogeal's Approximation Method

(B) Voage's Approximation Method

(C) Vangel's Approximation Method

(D) Vogel's Approximation Method

131. One can find initial basic feasible solution by using :

(A) VAM

(B) MODI

(C) Optimality test

(D) None of these

132. Once the initial basic feasible solution has been computed, what is the next step in the problem ?

(A) VAM

(B) Modified distribution method

(C) Optimality test

(D) None of these

133. If the total supply is less than the total demand, a dummy source (row) is included in the cost matrix with :

(A) Dummy Demand

(B) Dummy Supply

(C) Zero Cost

(D) Both (A) and (B)

134. Let $L^p[a, b]$, the class of all p-integrable functions over [a, b]. Let p and q be non-negative extended real numbers such that $\frac{1}{p}+\frac{1}{q}=1$. If $f \in L^p[a, b]$ and $g \in L^q[a, b]$, then :

(A) $fg \in L^1[a, b]$

(B) $fg \in L^p[a, b]$

(C) $fg \in L^q[a, b]$

(D) None of these

135. Let $0 < p < 1$ and $f, g \in L^p[a, b]$, such that $f \geq 0$ and $g \geq 0$. Then :

(A) $\| \mathbf{f} - \mathbf{g} \|_p \leq \| \mathbf{f} \|_p + \| \mathbf{g} \|_p$

(B) $\| \mathbf{f} + \mathbf{g} \|_p \geq \| \mathbf{f} \|_p + \| \mathbf{g} \|_p$

(C) $\| \mathbf{f} + \mathbf{g} \|_p \neq \| \mathbf{f} \|_p + \| \mathbf{g} \|_p$

(D) None of these

136. Let $L^\infty[a, b]$ space of measurable essentially bounded functions on $[a, b]$. Then which of the following is true ?

(A) $L^p[a, b] \subset L^\infty[a, b]$, $1 \leq p < \infty$

(B) $L^\infty[a, b] = L^p[a, b]$, $1 \leq p < \infty$

(C) $L^\infty[a, b] \subset L^p[a, b]$, $1 \leq p < \infty$

(D) None of these

137. The real linear space $C^1[0, 1]$ of all continuously differentiable functions defined on [0, 1] equipped with the norm given by :

$$\| X \|_\infty = \sup_{x \in [0,1]} | x(t) |$$

is an :

(A) Complete normed space

(B) incomplete normed space

(C) Not a normed space

(D) None of these

138. Let Y be a subspace of a normed space X. Then Y is complete implies that :

(A) Y is open

(B) Y is closed

(C) Y is bounded

(D) Y is semi-open

139. $L^p[a, b]$, $1 < p < \infty$, is :

(A) Not separable

(B) Separable sometime

(C) Separable

(D) Both separable and not separable

140. Let X and Y be normed spaces over the field $\mathbb{R}$ and $T : X \to Y$ be a linear operator. Then T is continuous if and only if :

(A) T is not bounded

(B) $\sup T = \infty$

(C) $\inf T = \infty$

(D) T is bounded

141. Let X and Y be Banach spaces over the field K and $T : X \rightarrow Y$ be a linear operator. Then T is closed if and only if :

(A) T is open

(B) T is unbounded

(C) T is bounded

(D) T is linear

142. Let X and Y be normed spaces and $T : X \rightarrow X$ be a linear operator. Then T has a closed graph, then :

(A) T has a closed null space

(B) T has a open null space

(C) T is one-one

(D) $T \neq I$

143. The space ℓ_p is a Hilbert space if and only if :

(A) p is even

(B) $p > 1$

(C) $p = \infty$

(D) $p = 2$

144. Dual space of $\ell^p(n)$, $1 < p < \infty$, is

(A) $\ell^p(n)$

(B) $\ell^q(n)$, $1 < q < \infty$, and $\frac{1}{p} + \frac{1}{q} = 1$

(C) $\ell^\infty(n)$

(D) $\ell^1(n)$

145. Let H is a Hilbert space and Y is a subspace of H. Then Y is complete if and only if :

(A) Y is closed in H

(B) Y is open in H

(C) Y is neither open and nor closed in H

(D) None of these

146. If $f(x) = \frac{1}{x}$, $1 < x < \infty$, then

(A) $f \in L^2(1, \infty)$ and $\| \mathbf{f} \| = 1$

(B) $f \in L^2(1, \infty)$ and $\| \mathbf{f} \| = 0$

(C) $f \notin L^2(1, \infty)$ and $\| \mathbf{f} \| = 1$

(D) $f \notin L^2(1, \infty)$ and $\| \mathbf{f} \| = 0$

147. If $f \in L^2(0, \infty)$ and $\lim_{x \to \infty} f(x)$ exists and is equal to :

(A) 1

(B) 0

(C) ∞

(D) None of these

148. A Banach space :

(A) Have a denumerable (Hamel) basis

(B) Cannot have a denumerable (Hamel) basis

(C) Is a normed linear space which is not complete

(D) None of these

149. A subset of a Hilbert space is weak bounded if and only if it is :

(A) Unbounded

(B) Bounded

(C) Empty

(D) None of these

150. Let $f : \mathbb{C} \to \mathbb{C}$ such that $f(z) = \overline{z}$ for all $z \in \mathbb{C}$. Then :

(A) f is non-analytic in $\mathbb{C}$

(B) f is analytic in $\mathbb{C}$

(C) Derivative of f exists only at $z = 0$

(D) None of these

151. Which of the following statement is correct ?

(A) If f(z) is continuous at z_0, then f(z) is analytic at z_0.

(B) If f(z) is analytic at z_0, then f(z) is continuous at z_0.

(C) If f(z) is analytic at z_0, then f(z) is not continuous at z_0.

(D) If f(z) is analytic at z_0, then $f(z) = 0$

152. If $f(z) = u(x, y) + iv(x, y)$ is analytic in the region Ω, then u and v are harmonic in Ω if :

(A) Both u and v are continuous in Ω

(B) Both u and v are differentiable in Ω

(C) They have continuous second order partial derivative in Ω

(D) Both u and v are piecewise continuous in Ω

153. Let $f : \mathbb{C} \to \mathbb{C}$ such that $f(z) = \dfrac{\sin\sqrt{z}}{\sqrt{z}}$ for all $z \in \mathbb{C}$. Then :

(A) $z = 0$ is not a removable singular point

(B) $z = 0$ is essential singular point

(C) $z = 0$ is a zero $f(z)$

(D) $z = 0$ cannot be a branch point

154. Let $f : \mathbb{C} \to \mathbb{C}$ such that $f(z) = \sin^{-1}\dfrac{1}{z}$ for all $z \in \mathbb{C}$, then f has :

(A) Essential singularity at $z = 0$

(B) Removable singularity at $z = 0$

(C) Pole at $z = 1$

(D) None of these

155. if f (z) be continuous in a simply-connected region Γ and suppose that $\oint_C f(z)dz = 0$ around a simple closed curve C in Γ, then :

(A) $f(z)$ is analytic in Γ

(B) $f(z) = 0$ in Γ

(C) $f(z)$ is constant in Γ

(D) $f(z)$ is not analytic in Γ

156. The value of $\oint_C \dfrac{dz}{z-a}$, where C is any simple closed curve and $z = a$ is inside C is given by :

(A) 2π

(B) $-2\pi i$

(C) $2\pi i$

(D) 0

157. Let $f : \mathbb{C} \to \mathbb{C}$ such that $f(z) = \sin z$ for all $z \in \mathbb{C}$. Then :

(A) $-1 \leq \sin z \leq 1$

(B) $\sin z$ is bounded

(C) $\sin z$ is constant

(D) $\sin z$ is not bounded in $\mathbb{C}$

158. If $f(z)$ is analytic inside and on a closed curve C and is not identically equal to a constant, then the maximum value of $|f(z)|$ occurs :

(A) In C

(B) At center of C

(C) On C

(D) None of these

159. If f(z) and g(z) are analytic inside and on a simple closed curve C and if | g(z) | < |f(z) | on C, then :

(A) f(z) – g(z) and f(z) have same number of zeroes inside C

(B) f(z),g(z) and f(z) have same number of zeroes inside C

(C) f(z) – g(z) = 0 inside C

(D) f(z) + g(z) and f(z) have same number of zeroes inside C.

160. If f(z) is analytic inside and on the boundary C of a simply-connected region Γ, then $\oint_C \frac{f(z)}{(z-a)^2} dz$ is :

(A) 0

(B) $2\pi i f'(a)$

(C) 1

(D) $2\pi i f(a)$

161. Every polynomial equation p(z) of degree n has exactly :

(A) n roots

(B) n – 1 roots

(C) More than n roots

(D) Less than n roots

162. If f(z) = u(x, y) + iv(x, y) is analytic in the region Ω, then u and v are satisfies :

(A) Wave equation

(B) Laplace equation

(C) Heat equation

(D) None of these

163. The equation z tan z = a, a > 0 has

(A) Exactly n roots

(B) Exactly n imaginary roots

(C) No roots

(D) Infinitly many roots but no imaginary roots

164. The value of $\int_0^\infty \frac{\sin x}{x} dx$:

(A) π

(B) ∞

(C) $\frac{\pi}{2}$

(D) $\frac{\pi}{4}$

165. If $w = f(z) = u(x, y) + iv(x, y)$ is analytic in the region Ω, then $\frac{\partial(u, v)}{\partial(x, y)}$ is equal to :

(A) $|f'(z)|^2$

(B) $f^2(z)$

(C) $f(z) f'(z)$

(D) 0

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SPACE FOR ROUGH WORK

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