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Subject Code No. : 38 **Sl. No. : 0802**

TEST BOOKLET

LECTURERS IN NON-GOVERNMENT AIDED COLLEGES

MATHEMATICS

Time Allowed : 2 Hours *Maximum Marks : 100*

: INSTRUCTIONS TO CANDIDATES :

1. IMMEDIATELY AFTER THE COMMENCEMENT OF THE EXAMINATION, YOU SHOULD CHECK THAT THIS TEST BOOKLET **DOES NOT** HAVE ANY UNPRINTED OR TORN OR MISSING PAGES OR ITEMS ETC. IF SO, GET IT REPLACED BY A COMPLETE TEST BOOKLET OF THE SAME SERIES ISSUED TO YOU.

2. You have to enter your **Roll No.** on the Test Booklet in the Box provided alongside. **DO NOT** write *anything else* on the Test Booklet.

3. This Test Booklet contains **100** items (questions). Each item (question) comprises four responses (answers). You have to select the correct response (answer) which you want to mark (darken) on the Answer Sheet. In case, you feel that there is more than one correct response (answer), you should mark (darken) the response (answer) which you consider the best. In any case, choose **ONLY ONE** response (answer) for each item (question). If more than one response is darkened it will be considered as wrong answer.

4. You have to mark (darken) all your responses (answers) **ONLY** on the **separate Answer Sheet** provided, by **using BALL POINT PEN (BLACK)**. See instructions in the Answer Sheet.

5. All items (questions) carry equal marks. All items (questions) are compulsory. Each wrong response will result in negative marking of **0.25** mark.

6. Before you proceed to mark (darken) in the Answer Sheet the responses to various items (questions) in the Test Booklet, you have to fill in some particulars in the Answer Sheet as per the instructions in your **Admission Certificate**.

7. After you have completed filling in all your responses (answers) on the Answer Sheet and after conclusion of the examination, you should hand over to the Invigilator the *Answer Sheet* issued to you. You are allowed to take with you the candidate's copy/second page of the Answer Sheet along with the *Test Booklet* after completion of the examination for your reference.

Candidate's full signature **Invigilator's signature**

CO – 17/18

2016

(Turn over)

UNIT – I

1. Find out the rank of the matrix :

$$\begin{bmatrix} 1 & a & b & . & . & . & . & p \\ 1 & a & b & . & . & . & . & p \\ . & . & . & & & & & \\ . & . & . & & & & & . \\ . & . & . & . & . & . & . & . \\ . & . & . & & & & & . \\ 1 & a & b & . & . & . & . & p \end{bmatrix}$$

(A) 0

(B) – 1

(C) 1

(D) p

2. The following system of linear equations has :

$x - 2y + z = 0$

$2x - y + 2z = 0$

(A) Unique solution

(B) Infinite solution

(C) Finite solution

(D) No solution

3. The Eigenvectors corresponding to distinct eigenvalues of a matrix are :

(A) Orthogonal

(B) Linearly dependent

(C) Linearly independent

(D) None of these

4. If 1, 1, 5 are the eigenvalues of the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$, then what are the eigenvalues of A^2 ?

(A) 1, – 1, 5

(B) 0, 1, 1

(C) 0, 0, 3

(D) 1, 1, 25

5. Which of the following set of vectors is linearly independent ?

(A) {(2, 3, 1), (2, 1, 3), (1, 1, 1)}

(B) {(0, – 1, 3), (3, 4, 3), (1, 1, 2)}

(C) {(2, 3, 1), (1, 0, 1), (1, 1, 2)}

(D) All of these

6. Let $f : A \rightarrow B$ is mapping from set A to set B, and f is one-one and onto. Then $f^{-1} : B \rightarrow A$ is :

(A) One-one and onto

(B) One-one but not onto

(C) Not one-one, but onto

(D) Neither one-one nor onto

7. The power set of $A = \{\phi, \{\phi\}, \{\{\phi\}\}, 5\}$ consists of :

(A) 16 elements

(B) 15 elements

(C) 32 elements

(D) None of these

8. The order of a subgroup of a group G of order 19 is :

(A) 1

(B) 19

(C) 1 or 19

(D) ≤ 19

9. Which of the following is not correct ?

(A) Every cyclic group is an abelian group

(B) Every group of odd order is cyclic

(C) The order of a cyclic group and the order of its generating element is equal.

(D) Every subgroup of a cyclic group is cyclic

10. Let G be a group of order 10. Then :

(A) All proper subgroups of G are abelian

(B) Some but not all proper subgroups of G are abelian

(C) No proper subgroup of G is abelian

(D) All proper subgroups of G, with only even order, are abelian

11. The number of generators of a cyclic group of order 6, is :

(A) 6

(B) 2

(C) 3

(D) 1

12. Let $S = \{1, 2, 3, 4\}$ and $A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 4 & 2 \end{pmatrix}$, then $\mathbf{A}^{-1}$ is :

(A) An even permutation

(B) An odd permutation

(C) Not a permutation

(D) An identity permutation

13. Let $G = \{Z \mid Z \in C, |Z| = 1\}$, C : complex numbers. Then which of the following is true ?

(A) G doesn't form a semigroup

(B) G doesn't form a monoid

(C) G doesn't form a group

(D) None of these

14. The number of algebraic terms in expansion of a determinant of order 5 is :

(A) 24

(B) 120

(C) 60

(D) 121

15. If α, β, γ are roots of $x^3 - x - 1 = 0$, the equation whose roots are $\frac{1+\alpha}{1-\alpha}, \frac{1+\beta}{1-\beta}, \frac{1+\gamma}{1-\gamma}$ is :

(A) $y^3 + 7y^2 - y + 1 = 0$

(B) $y^3 + y^2 + 7y - 1 = 0$

(C) $y^3 - 7y^2 + 7y + 1 = 0$

(D) $y^3 - 7y^2 + 1 = 0$

16. The number of positive roots of the equation f(x) = 0 is :

(A) Equal to no. of changes of sign in f(x)

(B) $\leq$ no. of changes of sign in f(x)

(C) $\geq$ no. of changes of sign in f(x)

(D) = no. of changes of sign in f(–x)

17. The condition that the sum of two roots α, β of $x^4 + p_1x^3 + p_2x^2 + p_3x + p^4 = 0$, zero is :

(A) $p_1p_4 - p_1p_2^2p_4 + p_3 = 0$

(B) $p_2^2p_4 - p_2p_3p_4 + p_1^2 = 0$

(C) $p_1p_3 - p_2^2p_3p_4 + p_2 = 0$

(D) $p_1^2p_4 - p_1p_2p_3 + p_3^2 = 0$

18. If p is a prime and a is any number prime to p, then p divides :

(A) $a^p - 1$

(B) a^{p-1}

(C) $a^{p-1} - 1$

(D) $a^p + 1$

19. Given that $35x \equiv 4 \pmod{0}$ and $55x \equiv 2 \pmod{12}$, the general value of x is :

(A) $x = 1 + 4t$

(B) $x = 14 + 36t$

(C) $x = 14 + 4t$

(D) $x = 1 + 36t$

20. If p is a prime, greater than 3, then for all integral values of m, $\underline{|mp} - \underline{|m} \cdot (\underline{|p})^m$ is divisible by :

(A) p^{m+3}

(B) p^m

(C) p^{m-3}

(D) p^{m+1}

UNIT – II

(REAL ANALYSIS)

21. If $\{K\alpha\}_{\alpha \in \Delta}$ (where Δ is an index set) is a collection of compact subsets of a metric space X such that the intersection of every finite sub-collection of $\{K\alpha\}_{\alpha \in \Delta}$ is non empty, then the cardinality of the set $\bigcap_{\alpha \in \Delta} K\alpha$ is :

(A) 0

(B) 1

(C) ≥ 1

(D) $\leq i$

22. Let P be a nonempty perfect set in R^{2016}. Then P is :

(A) Finite

(B) Countable

(C) Uncountable

(D) $[0, a]^{2016}$, where $a \in R$

23. The set $A = \{m + n\sqrt{2016} : m, n \in Z\}$:

(A) Is dense in R

(B) Has no limit point

(C) Has limit points in $R \setminus Q$

(D) Has limit points in Q

24. If A be a set with cardinality α then the cardinality of the set of all subsets of A is :

(A) α

(B) 2^{α}

(C) 2α

(D) α^2

25. Let $\Sigma\alpha_n$ is a series of complex numbers which converges absolutely then every rearrangement of $\Sigma\alpha_n$:

(A) Coverges

(B) Diverges

(C) Converges and converges to the same sum

(D) Converges but not necessarily converges to the same sum

26. Which of the following statement is false ?

(A) Any product of compact spaces is compact

(B) Any product of Hausdörff spaces is Hausdörff

(C) Any product of connected spaces is connected

(D) Any product of metrizable spaces is metrizable

27. Let X = R with confinite topology. Then X is a :

(A) First countable space

(B) T_1 space

(C) Regular space

(D) Normal space

28. Suppose X, Y, Z are metric spaces and Y is compact. Let f map X into Y, let g be a continuous one-to-one mapping of Y into Z, and put h(x) = g(f(x)) for $x \in X$. If h is uniformly continuous then f is :

(A) Continuous

(B) Uniformly continuous

(C) May be discontinuous

(D) Strictly increasing

29. Let the function f define by $f(x) = \frac{x^3+1}{3}$, for $x \in R$ and setting $x_{n+1} = f(x_n)$ for $n \in N$. Then which of the following statement is correct ?

(A) If $0 < x_1 < \frac{3}{2}$, then $\lim x_n \in (0, 1)$

(B) If $0 < x_1 < \frac{3}{2}$, then $\lim x_n \in (1, 2)$

(C) If $x_1 < -2$, then $\lim x_n = +\infty$

(D) If $x_1 > 2$, then $\lim x_n = 3^{10}$

30. Let $\{A_n\}_{n \in N}$ be a sequence of connected subsets of R^2 such that $A_{n+1} \subset A_n$ for $n \in N$. Then the set $\bigcap_{\alpha \in \Delta} A_n$ is :

(A) Connected

(B) Disconnected

(C) May not be connected

(D) R

31. Suppose that f is real-valued function defined in an open set $E \subset R^n$ and that the partial derivatives D_1f, D_2f, D_3f, ..., D_nf are bounded in E. Then f is :

(A) Bounded

(B) Unbounded

(C) Continuous

(D) Uniformly continuous

32. The series $\sum_{k=1}^{n} \frac{x}{n(1+nx)^2}$ is :

(A) Convergent on [0, 1]

(B) Divergent on [0, 1]

(C) Uniformly convergent on [0, 1]

(D) Convergent on [0, 1] but not uniformly

33. The large interval in which $\sum_{n=1}^{n} (-1)^n \frac{x^n}{n}$ converges is :

(A) $(-1, 1]$

(B) $[-1, 1)$

(C) $(-1, 1)$

(D) $[-1, 1]$

34. Let a > 1 and for x > 0 define $f(x) = (x^{\alpha} - 1) + \alpha(1 - x)$, then :

(A) f is an increasing function on $(0, \infty)$

(B) f is decreasing function on $(0, \infty)$

(C) $f(x) \geq 0$ for all $x > 0$

(D) f takes both positive and negative values for $x > 0$

35. For $n \in N$, let $f_n : [0, 1] \to R$ be the function defined by $f_n(x) = \frac{1}{1 + e^{nx^2}}$.

Then $\lim_{n \to \infty} \int_0^1 f_n(x)\, dx = 0$ by :

(A) Fatou's Lemma

(B) Lebesgue dominated convergence theorem

(C) Fubini's theorem

(D) Positivity of each f_n

36. Let $f_n(x) = \begin{cases} \frac{1}{x} \text{ if } n < x < n+1 \\ 0 \text{ otherwise} \end{cases}$

If $f(x) = \lim_{n \to \infty} f_n(x)$, then $\int_0^\infty f(x)dx =$

$\lim_{n \to \infty} \int_0^\infty f_n(x)dx$ by :

(A) Bounded convergence theorem

(B) Monotone convergence theorem

(C) Dominated convergence theorem

(D) Fubini's theorem

37. Let A_k, k = 1, 2, 3, ..., n be pairwise disjoint subsets of real line R with Lebesgue measure $m(A_k) = k^3$. If $A = \bigcup_{k=1}^n A_k$ and $\phi(x) = \sum_{k=1}^n \frac{1}{k}$

$XA_k(x)$, $XA_k(x)$ is the characteristic function of A_k, then the Lebesgue integral $\int_A \phi(x)dx$ equals :

(A) 0

(B) log n

(C) $\frac{n(n+1)}{2}$

(D) $\frac{n(n+1)(2n+1)}{6}$

38. Let $f_n : [a, b] \to R$ be defined by

$f_n(x) = \begin{cases} 2n \text{ if } \frac{1}{2n} < x < \frac{1}{n} \\ 0 \text{ otherwise} \end{cases}$

Then the values of $\int_0^1 \lim_{n \to \infty} f_n d\mu$ and $\lim_{n \to \infty} \int_0^1 f_n d\mu$ (where μ is Lebesgue measure on R) are respectively :

(A) 0, 0

(B) 0, 1

(C) 1, 0

(D) 1, 1

39. Let $f : (0, 1) \to R$ be a bounded Riemann integrable function and let $g : R \to R$ be continuous Then $g \circ f$ is :

(A) Riemann integrable

(B) Continuous

(C) Lebesgue integrable, but not Riemann integrable

(D) Not necessarily measurable

40. Let S be a non-empty Lebesgue measurable subset of R such that every subset of S is measurable. Then the measure of S is equal to the measure of any :

(A) Subset of S

(B) Countable subset of S

(C) Bounded subset of S

(D) Closed subset of S

UNIT – III

41. The order of convergence of Newton-Raphson method is :

(A) 0

(B) 3

(C) 2

(D) 1

42. The maximum error associated with composite Simpson's rule is :

(A) $-\frac{h^5}{90} f^4(\zeta)$

(B) $-\frac{h^4}{180} (b-a) f^4(\zeta)$

(C) $-\frac{h^5}{180} (b-a) f^4(\zeta)$

(D) None of these

43. Which of the following is false ?

(A) Trapezoidal rule is 2 points quadrature

(B) Simpson's $\frac{1}{3}$ rule is 3 points quadrature

(C) Weddle's rule is 8 points quadrature

(D) All of these

44. The degree of precision of trapezoidal rule is :

(A) 1

(B) 2

(C) 3

(D) 4

45. The second approximation for the root of the equation $3x = \cos x + 1$ between 0 and 1 with $x_0 = 0{\cdot}6$ by Newton-Raphson's method is :

(A) 0·607

(B) 0·517

(C) 0·606

(D) 0·350

46. The value of $\int_2^3 \frac{dx}{1+2x}$ correct upto 3 decimal places by Simpson's $\frac{1}{3}$ rule is :

(A) 0·148

(B) 0·138

(C) 0·166

(D) 0·158

47. $\frac{dy}{dx} = x^2 + y$, $y(0) = 1$, the value of $y(0.4)$ by Euler's method is :

(A) 1·03

(B) 1·04

(C) 1·02

(D) 1·01

48. A curve passes through the points, given by the following table :

x	y
1	2
1·25	2·1
1·50	2·6
1·75	3
2	2·5
2·25	2·8
2·50	3.1
2·75	3·2
3	4

The area bounded by the curve, x-axis and the lines $x = 1$ and $x = 3$ is :

(A) 5·58

(B) 22·3

(C) 11·15

(D) None of these

49. The order of error is fourth order Runge-Kutta method is :

(A) 2

(B) 3

(C) 4

(D) 5

50. (n + 1) point Newton-Cote's formula had degree of precision :

(A) n, if n is odd and n + 1, if n is even

(B) n + 1, if n is odd and n, if n is even

(C) $n+1 \; \forall \, n$

(D) None of these

51. For the following table :

x	f(x)
0	41
1	43
2	47
3	53
4	61
5	71

the degree of the interpolating polynomial f(x) is :

(A) 6

(B) 3

(C) 5

(D) 2

52. If $L\{f(t)\} = \bar{f}(s)$, then $L\left\{\int_0^t f(u)du\right\} =$

(A) $\frac{1}{t}\bar{f}(s)$

(B) $\frac{1}{s}\bar{f}(s)$

(C) $\bar{f}(s)$

(D) $s\,\bar{f}(s)$

53. $L^{-1}\left\{\frac{1}{s^2+9}\right\} =$

(A) $\frac{1}{9}\sin 3t$

(B) $\frac{1}{3}\cos t$

(C) $\frac{1}{3}\sin 3t$

(D) $\frac{1}{3}\cos 3t$

54. The complete integral of the equation $2zx - px^2 - 2qxy + pq = 0$ by Charpit's method is given by (b is a constant) :

(A) $z = ay + b(x^2 - a)$

(B) $z = ax + (x^2 + b)$

(C) $z = y - a(x^2 - b)$

(D) $z = x + y^2 - a$

55. The partial differential equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = \frac{2z}{x}$ is satisfied by (ϕ is an arbitrary function) :

(A) $z = \frac{1}{x}\phi(x-y) + \frac{1}{y}\phi'(x-y)$

(B) $z = \frac{1}{x}\phi(y-x) + \phi'(y-x)$

(C) $z = \phi(y-x) + \frac{1}{y}\phi(y-x)$

(D) $z = \phi(y-x) + \phi'(x-y)$

56. The solution of the equation $z = \frac{1}{2}(p^2 + q^2)(p - x)(q - y)$ which passes through the x-axis is :

(A) $z = y(4x - y)$

(B) $z = \frac{1}{3}x(y + 4x)$

(C) $z = \frac{1}{2}y(4x - 3y)$

(D) $z = y(4x + 3y)$

57. The surface which intersect the surfaces of the system $z(x + y) = C(3z + 1)$ orthogonally and which passes through the circle $x^2 + y^2 = 1$, $z = 1$ is :

(A) $x^2 + y^2 + z^2 + 2 = z^3$

(B) $x^2 + y^2 = 2z^3 + z^2 - 2$

(C) $x^2 + y^2 + 2 = 0$

(D) $x^2 + y^2 = z^3 + 2$

58. Using Jacob's method, the solution of $(p^2 + q^2)x = pz$ is :

(A) $z = a\, x^{\frac{1}{a}} y^a$

(B) $z = bx^a y^a$

(C) $z = b\, x^{\frac{1}{a}} y^{\frac{1}{a}}$

(D) $z = bx^a y^{\frac{1}{a}}$

59. The solution of the equation $(x^2D^2 + xyDD' - 2y^2D'^2 - xD - 6yD')\,z = 0$ is :

(A) $z = \phi_1\left(\frac{y}{x}\right) + \phi_2\left(\frac{y}{x^2}\right)$

(B) $z = \phi_1(y^x) + \phi_2\left(\frac{x}{y}\right)$

(C) $Z = \phi_1\left(\frac{y}{x^2}\right) + \phi_2(xy)$

(D) None of the above

60. $\Delta^2 \cos 2x =$

(A) $4 \sinh \cos (x + h)$

(B) $-4 \sin x \cosh (x + h)$

(C) $4 \sin^2 h \cos (x + h)$

(D) $-4 \sin^2 h \cos 2(x + h)$

UNIT – IV

61. The Boolean function $F = A . B' . C + A' \cdot B \cdot C + A' \cdot B \cdot C' + A \cdot B' \cdot C'$ is equivalent to :

(A) $B \oplus C$

(B) $A \oplus B$

(C) $C \oplus A$

(D) $A \oplus B \oplus C$

62. Let B be a Boolean algebra and $a, b \in B$. Then $a \cdot b' = 0$ iff :

(A) $a \cdot b = b$

(B) $a \cdot b = 1$

(C) $a \cdot b = a$

(D) $a \cdot b = 0$

63. The number of terms in a complete DNF of a Boolean function with n variables is :

(A) (n) !

(B) $> 2^n$

(C) $< 2^n$

(D) None of these

64. Let L be a Lattice, then $a \wedge b = a$ iff :

(A) $a \vee b = b$

(B) $a \vee b = a$

(C) $a \vee b = \phi$

(D) None of these

65. The Hasse diagram of a finite linearly ordered set is always :

(A) A triangular structure

(B) A square structure

(C) A linear path

(D) None of these

66. Let $(L, \leq)$ be a poset. If $(L, \leq)$ is a lattice, then the dual of the poset $(L, \leq)$ is :

(A) Also a lattice

(B) Not a lattice

(C) May or may not be a lattice

(D) None of these

67. The power set of any universal set is :

(A) Bounded lattice

(B) Unbounded Lattice

(C) Only upper bounded lattice

(D) Only lower bounded lattice

68. $P \vee (P \rightarrow Q) \vee \{\sim (P \vee Q)\}$ is a :

(A) Tautology

(B) Contradiction

(C) Contingency

(D) Satisfiable but invalid

69. The length of a Hamiltonian path (if it exists) in a connected graph of n vertices is :

(A) n

(B) $n + 1$

(C) $n - 1$

(D) $\frac{n(n+1)}{2}$

70. The number of spanning trees in a complete graph with 5 vertices is :

(A) 125

(B) 25

(C) 625

(D) 120

71. The maximum number of edges possible in a simple bipartite graph with 12 vertices is :

(A) 24

(B) 36

(C) 48

(D) 64

72. The total number of vertices in a graph containing 21 edges, where 3 vertices are of degree 4 and other vertices are of degree 3, is :

(A) 10

(B) 13

(C) 11

(D) 12

73. A given connected graph G is an Euler graph, iff all vertices of G are of :

(A) Same degree

(B) Even degree

(C) Old degree

(D) Different degree

74. Suppose a planar graph has K_1 components, e_1 edges, v_1, vertices and r_1 regions. Then :

(A) $r_1 = e_1 - v_1 + k_1 - 1$

(B) $r_1 = e_1 - v_1 + k_1 + 1$

(C) $r_1 = e_1 + v_1 + k_1 - 1$

(D) $r_1 = e_1 + v_1 + k_1 + 1$

75. The degree of each region in a polyhedral graph (degree of each region ≥ 3) with 12 vertices and 30 edges is :

(A) 2

(B) 4

(C) 3

(D) 5

76. A positive semi-definite quadratic form $f(x) = X^T AX$, where $X^T \in R^n$ and A is an n-rowed square matrix, is a :

(A) Strictly convex function over R^n

(B) Strictly concave function over R^n

(C) Convex function over R^n

(D) Concave function over R^n

77. The two phase simplex method is used to solve a Linear Programming Problem which involves :

(A) Slack variable

(B) Basic variable

(C) Surplus variable

(D) Artificial variable

78. Let f(x, y) be such that both $\min_y \max_x f(x, y)$ and $\max_x \min_y f(x, y)$ exist. Then necessary and sufficient condition for the existence of a saddle point (x_0, y_0) of f(x, y) is that :

(A) $f(x_0, y_0) = \min_x \max_y f(x, y)$

(B) $f(x_0, y_0) = \max_x \min_y f(x, y)$

(C) $\min_x \max_y f(x, y) = \max_x \min_y f(x, y)$

(D) $f(x_0, y_0) = \min_y \max_x f(x, y) = \max_x \min_y f(x, y)$

79. The two person zero-sum game which has a skew symmetric pay off matrix, has a value :

(A) 1

(B) – 1

(C) ∞

(D) 0

80. There exists a finite optimal solution to a L. P. P. iff there exists :

(A) A feasible solution to its dual

(B) A bounded solution to its dual

(C) A feasible solution to both the primal and its dual

(D) A feasible solution to the primal

UNIT – V

81. Let f be a nonconstant entire function. Which of the following properties is possible for f for each $z \in \mathbb{C}$?

(A) $\operatorname{Re} f(z) = \operatorname{Im} f(z)$

(B) $\operatorname{Im} f(z) < 0$

(C) $|f(z)| < 1$

(D) $f(z) \neq 0$

82. Let $f(z) = a_0 + a_1 z + a_2 z^2 + \cdots + a_n z^n$ and $g(z) = b_1 z + b_2 z^2 + \cdots + b_n z^n$ be complex polynomials. If a_0, b_1 are non-zero complex numbers, then the residue of f(z) / g(z) at z = 0 is equal to :

(A) $\frac{a_0}{b_1}$

(B) $\frac{b_1}{b_0}$

(C) $\frac{a_1}{b_1}$

(D) $\frac{a_0}{a_1}$

83. For the function $f(z) = \dfrac{z^8 + z^4 + 2}{(z-1)^3\,(3x+2)^2}$ which one of the following is true ?

(A) $z = -\frac{3}{2}$ is a pole of order 2

(B) $z = 1$ is a pole of order 4

(C) $z = 0$ is an isolated essential singularity

(D) $z = \infty$ is a pole of order 3

84. Let $f(z) = \dfrac{\sin z}{z^2} - \dfrac{\cos z}{z}$ then :

(A) f has a pole of order 2 at $z = 0$

(B) f has a simple pole at $z = 0$

(C) $f_{|z|=1} f(z)dz = 0$

(D) The residue of f at $z = 0$ is $2\pi i$.

85. The minimum possible value of $|z|^2 + |z-3|^2 + |z-6i|^2$, where z is a complex number, is :

(A) 15

(B) 45

(C) 30

(D) 20

86. Let $f : D \to D$ be a holomorphic function with $f(0) = 0$ where $D = \{z \in C : |z| < 1\}$. Then :

(A) $|f'(0)| = 1$

(B) $|f(\frac{1}{2})| \leq \frac{1}{2}$

(C) $|f(\frac{1}{2})| \leq \frac{1}{4}$

(D) $|f'(0)| \leq \frac{1}{2}$

87. Consider the function $f, g : C \to C$ defined by $f(z) = e^z$, $g(z) = e^{iz}$. Let $S = \{z \in C : \text{Re } z \in [-\pi, \pi]\}$. Then :

(A) f is an onto entire function

(B) f is bounded on S

(C) g is a bounded function on C

(D) g is bounded on S

88. Let p(z), q(z) be two nonzero complex polynomials. Then $p(z)\overline{q(z)}$ is anlytic if and only if :

(A) p(z) is constant

(B) q(z) is constant

(C) p(z)q(z) is constant

(D) $\overline{p(z)}q(z)$ is constant

89. Let $\neq f : C \to C$ be analytic except for a simple pole at $z = 0$ and let $g : C \to C$ be analytic. Then the value of

$$\frac{\text{Res}_{z=0}\{f(z)g(z)\}}{\text{Res}_{z=0}\, f(z)} \text{ is :}$$

(A) $g(0)$

(B) $g'(0)$

(C) $\lim_{z \to 0} zf(z)$

(D) $\lim_{z \to 0} z\,f(z)\,g(z)$

90. Let $P(x)$ be a polynomial of real variable x of degree $k \geq 1$. Consider the power series $f(z) = \sum_{n=0}^{\infty} P(n)z^n$ where z is a complex variable. Then the radius of convergence of $f(z)$ is :

(A) 0

(B) 1

(C) k

(D) ∞

91. The radius of convergence of the power series $\sum_{n=0}^{\infty} (4n^4 - n^3 + 3)z^n$ is :

(A) 0

(B) 1

(C) 5

(D) ∞

92. If $|z_1| = |z_2| = \cdots = |z_n| = 1$, then the value of $|z_1 + z_2 + \cdots + z_n|$ is :

(A) 1

(B) $|z_1| + |z_2| + \cdots + |z_n|$

(C) $\left|\frac{1}{z_1} + \frac{1}{z_2} + \cdots + \frac{1}{z_n}\right|$

(D) $n + 1$

93. Let f, g be meromorphic functions on C. If f has a zero of order k at $z = a$ and g has a pole of order m at $z = 0$, then $g(f(z))$ has :

(A) A pole of order km at $z = a$

(B) A zero of order km at $z = a$

(C) A pole of order $|k - m|$ at $z = a$

(D) A zero of order $|k - m|$ at $z = a$

94. If X and Y be normed linear spaces and $T : X \rightarrow Y$ be a linear transformation, the kernal of T is defined as $Ker(T) = \{x \in X : T(x) = 0)$ then :

(A) Ker(T) is open if T is continuous

(B) Ker(T) is open if T is bounded

(C) Ker(T) is closed if T is continuous

(D) Ker(T) is closed if T is bounded

95. Let X be a non-zero normed linear space. Then X is a Branch space if and only if :

(A) $\{x \in X : ||x|| \leq 1\}$ is complete

(B) $\{x \in X : ||x|| \geq 1\}$ is complete

(C) $\{x \in X : ||x|| = 0\}$ is complete

(D) $\{x \in X : ||x|| < 0\}$ is complete

96. Let H be a Hilbert space, and T_1, T_2 be two adjoint operators. Which one of the following is false ?

(A) $(T_1 + T_2)^* = T_1^* + T_2^*$

(B) $(aT_1)^* = aT_1^*$

(C) $(T_1T_2)^* = T_2^*T_1^*$

(D) $(aT_1)^* = \bar{a}T_1^*$

97. If F is a closed set and Y is compact set then :

(A) $(F \cap Y)^c$ is closed

(B) $F \cap Y$ is open

(C) $F \cap Y$ is compact

(D) $F \cap Y$ is connected

98. Which one of the following is not a compact set if d is a usual matrix ?

(A) [0, 1]

(B) (0, 1)

(C) (X, d), where X is a finite

(D) $(\frac{1}{2}, \frac{1}{3})$

99. The dual space of ℓ_p is ℓ_q for :

(A) $1 \leq p < \infty$ and $1/p + 1/q = 1$

(B) $1 < p \leq \infty$ and $1/p + 1/q = 1$

(C) $1 \leq p \leq \infty$ and $1/p + 1/q = 1$

(D) $1 < p < \infty$ and $1/p + 1/q = 1$

100. In an inner product space which of the following is true ?

(A) $\|x + y\|^2 + \| x - y \|^2 = \| x \|^2 + \| y \|^2$

(B) $2(\|x + y\|^2 + \| x - y \|^2) = \| x \|^2 + \| y \|^2$

(C) $\|x + y\|^2 + \| x - y \|^2 = 2(\| x \|^2 + \| y \|^2)$

(D) $\|x + y\|^2 - \| x - y \|^2 = 2(\| x \|^2 + \| y \|^2)$

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